

Foreign Exchange Options

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Abstract. The Article deals with pricing and hedging of Foreign Exchange Options from a trader's perspective. We focus on First Generation Exotic Options and will give some examples on Second Generation Exotic Options. We will split up some of these options into their individual components and point out the risk arising from this type of options, as well as show ways to minimise these risks. Market conventions in Plain Vanilla and Exotic Options are described and deviations resp. exceptional features in these markets are shown.

1. Market Overview

More and more, the importance of Foreign Exchange Options for risk management and directional trades is recognised by companies and investors. Various Banks have been adapting their products to this situation during the past years. Different risk and profit profiles can be generated with Plain Vanilla or Exotic Options as individual products, as well as in combination of various products such as Structured Products. Financial engineers call this "playing with LEGO- bricks". Linear combinations of basic products are used to build Structured Products. In order to price Plain Vanilla or Exotic Options and show their risk, many professional trading systems were introduced and are being continuously developed.

With these systems, the traders are able to evaluate the positions in the individual currency pair or in currency portfolios at any time. In the FX Options market, Options trading systems such as *Fenics*, *Murex* or *Superderivates* are used. Some Banks started developing and using systems of their own, due to the very rapid development in this sector.

In order to comply with various customer requests, a successful trading desk in the interbank market is essential. This is mostly plain volatility trade. The market risk of short-term FX Options trading desk consists of changes in spot, volatility and interest rates. Since spot risk is easily eliminated by [delta hedging](#) and the effect of rates is small compared to the risk of changing volatility in the short term up to two years, managing volatility risk is the main task of the trader. Since the relationship of volatility and price of call or put options is monotone, it is equivalent to quote the price of an option either by the price itself or by the volatility implied by the [Black-Scholes Formula](#). The established market standard is quoting this implied volatility, which is why it is often viewed as a traded quantity. In the case of Plain Vanilla Options, a [Vega](#) long position is given when buying Plain Vanilla Options (Call or Put), conversely a Vega short position is given when selling Plain Vanilla Options. Volatility difference between Call and Put with same expiry and same deltas is called a risk reversal. If the risk reversal is positive, the market is willing to pay more for calls than for puts, if the risk reversal is negative, the market favours put options. The Butterfly measures the convexity of the “smile” of the volatility, i.e., the volatility for the out of the money and the in the money options, see [Foreign exchange market terminology](#) for details.

If the [delta hedge](#) is done with an interbank partner at the same time the option is traded, the trader can focus on the Vega position in his book. The [delta hedge](#) neutralizes the change of the option price caused by changes of the underlying. For long-term options with an expiry longer than 2 years or options with high interest rate sensitivity, the [delta hedge](#) should be replaced by a Forward Hedge, as the risk of interest rate sensitivity is mostly higher than the

volatility risk in this case. This means that instead of neutralizing spot risk by trading in the spot market, one would trade a forward contract with maturities matching those of the cash-flows of the option. This would simultaneously take care of the spot and the rate risk.

2. First Generation Exotic Options

First Generation Exotic Options are all options beyond Plain Vanilla Options that started trading in the 1990s, in particular barrier options, digital and touch products, average rate or Asian options, lookback and compound options. There is no strict separation between First and Second Generation Exotics as the viewpoint on what is first and second varies by the person in charge. Exotic Options are traded live in currency trading as opposed to Plain Vanilla Options, which mostly trade through automated systems. Trading Exotic Options is done by quoting the bid- and ask price of the product rather than the corresponding volatility, because the monotone relationship of volatility and price is often not guaranteed. When asking for a quote, the spot reference level is agreed upon at which the option is calculated and priced. This allows comparing quotes of the Exotic Options and is also the basis of the [delta hedge](#). In order to keep the [vega](#) risk low when fixing a deal, a Vega hedge can be done with the partner. In this case, Plain Vanilla Options (Calls/ Puts, ATM Straddles) are traded in order to offset the Vega of the Exotic Option. The default vega hedge is done with a straddle – an out-of-the-money call and an out-of-the-money put --, the reason being that this product does not have any delta, so one offsets the vega position without touching the delta position. Normally during the lifetime of the option, the risk is hedged dynamically across the entire option book.

The quoting bank (Market Maker) is the calculation agent. It stipulates the regulations under which pre-defined triggers are reached or how often the underlying is traded in certain pre-defined ranges. The Market Maker informs the Market User about the trigger event.

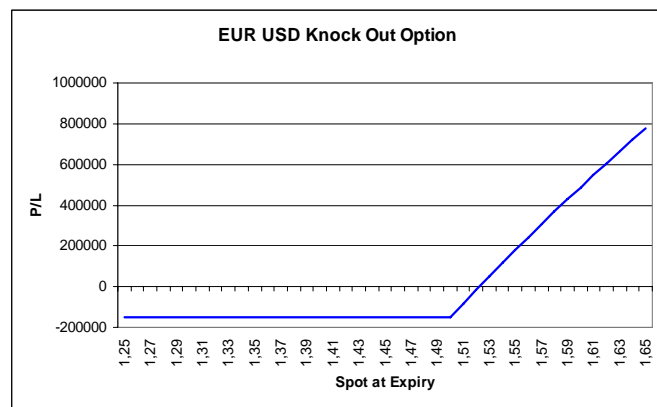
2.1. Barrier Options

Barrier Options are Vanilla Put and Call Options with the additional barriers. In case of a knock-out, the option expires worthless, if the spot ever trades at or beyond the pre-specified barrier. In case of a knock-in option, the option is only activated if the spot ever trades at or beyond the pre-specified barrier. The barrier is valid at all times between inception of the trade and the maturity time of the option.

One can further distinguish regular barrier options, where the barrier is out of the money and reverse barrier options, where the barrier is in-the-money.

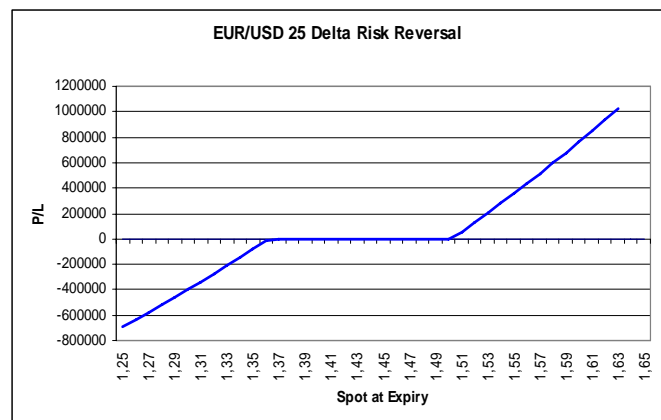
A regular Knock Out Barrier Option can basically be priced and semi-statically hedged by a risk reversal (Lego-brick principle).

An Example: EUR-USD Spot 1.4600
Expiry 6 Months, Strike 1.5000, EUR
CALL with regular Knock Out Trigger
at 1.4300.



Graph: Payoff-Profile of a EUR USD KO Option which is not knocked out during its lifetime

Hedging a short regular knock-out EUR call, we can go long a vanilla EUR Call with the same strike and the same expiry and go short a vanilla EUR Put with a strike such



Graph: Payoff-Profile of the EUR USD Risk Reversal at Expiry

that the value of the hedge portfolio is zero if the spot is at the barrier. The long call and short put is called a risk reversal and its Market price can be used as a proxy for the price of the regular knock-out call. In our example, it would be 1.3650 EUR Put. If the trigger is not reached, then the put expires worthless and the call offsets the knock-out call payoff. If the trigger is reached, the risk reversal can be cancelled with approximately zero value. The delta of a knock-out option is higher than the delta of the corresponding Plain Vanilla Option and is higher, the closer the trigger is to the underlying spot.

Reverse-Knock-Out and Reverse-Knock-In are more difficult to price and hedge as the risk profile of these options is difficult to replicate with other options. In this case, the trigger is in the money. The volatility risk of first and second order arising from these options can be hedged dynamically with risk reversals and butterflies (see [Vanna-Volga-Pricing](#)). However, all sensitivities take extreme values when getting closer to the trigger and closer to maturity. Delta positions can be a multiple of the notional amount. Therefore, it is difficult for the trader to perform dynamic hedging strategies. In order to manage these risks, short term reverse knock-out barrier options are often removed from the global books and are matched as individual positions, or are closed 2 to 3 weeks before expiry. The risk surcharge paid in this case is often smaller than the cost of keeping to such positions and hedge them individually.

2.2 Modifications and Extensions of Barrier Options

Standard extensions of barrier options are **double-barrier options**, where there is a barrier above and below the current spot. A double-knock-out option expires worthless if any of the two barriers are ever touched or crossed. A double-knock-in option only becomes a vanilla option if at least one of the two barriers is touched or crossed in the underlying.

A further modification of barrier options is the so-called **KIKO option** (knock-in-knock-out). This option can knock out at any time, however, must knock in to become alive. A short KIKO option can be statically hedged with a long Knock Out Option and a short Double Knock Out Option, if the spot value is between the triggers, and with a long Knock Out Option and a short Knock Out Option, if the spot value is above both triggers (LEGO brick principle).

Additional modifications of Barrier Options are **Window Barriers (Partial Barriers)**. In case of a window barrier option the trigger is valid only within a certain period of time. Commonly, this period of time is from inception of the trade until a specific date (*early ending*) or from a specific date during validity until expiry date of the option (*deferred start*). Arbitrary time intervals are possible.

For **European Barrier Options** the triggers are only valid at maturity. They can be statically hedged with Plain Vanilla Options and European Digital Options (LEGO brick principle).

2.3 Binary Options / Digital Options

Digital or binary options pay a fixed amount in a currency to be specified if the spot trades at or beyond a pre-specified barrier or trigger. For European Digitals the trigger is valid only at maturity, whereas for American Digitals the trigger is valid during the entire lifetime of the trade. In FX-interbank trade American Digitals are also called **One-Touch** (if the fixed amount is paid at maturity) or **Instant One-Touch** (if the fixed amount is paid at first hitting time). Further Touch Options are the so-called **No-Touch Options**, **Double-No-Touch Options** and **Double-One-Touch Options**. A no-touch pays only if the spot never touches or crosses the pre-specified trigger. A double-no-touch pays only if neither the upper trigger nor the lower trigger is ever touched or crossed during the lifetime of the contract. A double-one-

touch pays only if at least one of the upper or the lower trigger is touched. When buying a Double-No-Touch Option, a Vega short position is generated. This means that Double-No-Touch Options are cheap in phases of high volatility.

European Digital Options can be replicated with Bull or Bear Spreads with large amounts. Their market price can thus be approximated by liquid vanilla options. However, this type of option is difficult to hedge as the delta hedge close to expiry is zero almost everywhere.

3. General features when pricing Exotic Options

Most commercial software packages calculate the “theoretical value (TV)” of the Exotic Options, which is the value of the product in a [Black-Scholes Model](#) with constant parameters.

Knowing the TV is important for trading partners as it serves as a checksum to ensure that both parties talk about the same product. The market value, however, often deviates from this value because of so-called overhedge costs, which arise when hedging the exotic option. Every trader must be aware of the risk arising from these options and is able to control this risk dynamically in his book via the Greeks (price sensitivity with respect to market and model parameters). If a gain is generated by performing this hedge the price of an Exotic Option must be higher than the TV. Conversely, if the hedge leads to a loss the market price of the exotic option should be above TV.

A very important issue when trading Exotic Options is placing automatic spot orders at spot levels that could lead to a knock-out or expiry of the option. This order eliminates the [delta hedge](#) of the option automatically when reaching the trigger. This explains the occasional very heavy spot movements during specific trigger events in the market.

The following Vega structure is often found in options books as it stems from most of the [Structured Products](#) offered today in the FX-range: ATM vega long and Wing vega short. This is the reason for a long phase of low volatility and high butterflies for the past years, see also the section on [Foreign exchange smile: conventions and empirical facts](#).

4. Second Generation Exotic Options

We consider every exotic option as Second Generation if it is not a vanilla and not a first generation product.

Some of the common examples in Foreign Exchange Markets are **Range Accruals** and **Faders**.

A range accrual is a sum of digital call spreads and pays an amount of a pre-specified currency that depends on the number of currency fixings that come to fall inside a pre-specified range. A fader is any basic option product like a vanilla or barrier option, whose notional amount depends on the number of currency fixings that come to fall inside a pre-specified range. We distinguish fade-in products, where the notional grows with each fixing inside the range and fade-out products, where the notional decreases with each fixing inside the range.

Further extensions are Target Redemption Products, whose notional amount increases until a certain gain is reached. A common example is a **Target Redemption Forward (TRF)**. We provide a description and an example here: We consider a TRF in which a client sells EUR and buys USD at a much higher rate than current spot or forward rates. The key feature in this product is that client has a total target profit that, once hit, knocks out all future settlements (in the example below, all weekly settlements), locking the gains registered until then.

The idea is to place the strike over 5.5 big figures above spot to allow the client to quickly accumulate profits and have the trade knocked out after 5 or 6 weeks. The client will start

losing money if EURUSD starts fixing above the strike. On a spot reference of 1.4760 consider a one year TRF, in which the client sells 1 EUR 1 million per week at 1.5335, subject to a KO condition: if the sum of the client profits reaches the target, all future settlements are cancelled. We let the target be 0.30 (i.e. 30 big figures), measured weekly as $\text{Profit} = \text{Max}(0, 1.5335 - \text{EUR-USD-Spot-Fixing})$. As usual this type of forward is also traded at zero cost.

Example:

1st week fix	1.4800	Profit = 0.0535	Max(1.5335 - 1.4800 , 0)
2nd week fix	1.4750	Profit = 0.0585	Accumulated Profit = 0.1120
3rd week fix	1.4825	Profit = 0.0510	Accumulated Profit = 0.1630
4th week fix	1.4900	Profit = 0.0435	Accumulated Profit = 0.2065
5th week fix	1.4775	Profit = 0.0560	Accumulated Profit = 0.2625
6th week fix	1.4850	Profit = 0.0485	Accumulated Profit = 0.3110

Profit is capped at 0.30, so the client only accumulates the last 3.75 big figures and the trade knocks out.

Each forward will be settled physically every week until trade knocks out (if target is reached)

Another popular FX product is the **Time Option**, which is essentially a Forward Contract of American style, i.e. the buyer is entitled and obliged to trade a pre-specified amount at a pre-specified strike, but can choose the time within a pre-specified time interval.

The market is likely to continue to develop fast. **Besides Bermudan Style Options**, where early exercise is allowed at certain pre-specified times, [Basket Options](#) and the corresponding

structures are very much in demand in the market. Hybrid structures are exotic options whose payoff depends on underlying spots across different market sectors. We refer the reader to [1].

References:

[1] Wystup, U. (2006): FX Options and Structured Products, Wiley.

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