Vanna-Volga Pricing

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1 Vanna-Volga Pricing

The vanna-volga method, also called the *traders' rule of thumb* is an empirical procedure that can be used to infer an implied-volatility smile from three available quotes for a given maturity. It is based on the construction of locally replicating portfolios whose associated hedging costs are added to corresponding Black-Scholes prices to produce smile-consistent values. Besides being intuitive and easy to implement, this procedure has a clear financial interpretation, which further supports its use in practice. In fact, SuperDerivatives has implemented a type of this method in their pricing platform, as one can read in the patent that SuperDerivatives has filed. The VV method is commonly used in foreign exchange options markets, where three main volatility quotes are typically available for a given market maturity: the delta-neutral straddle, referred to as at-the-money (ATM); the risk reversal for 25 delta call and put; and the (vega-weighted) butterfly with 25 delta wings. The application of vanna-volga pricing allows us to derive implied volatilities for any options delta, in particular for those outside the basic range set by the 25 delta put and call quotes. The notion of risk reversals and butterflies is explained in Section ?? on FX market terminology.
In the financial literature, the vanna-volga approach was introduced by Lipton and McGhee (2002) in [2], who compare different approaches to the pricing of double-no-touch options, and by Wystup (2003) in [5], who describes its application to the valuation of one-touch options.

The vanna-volga procedure is reviewed in more detail and some important results concerning the tractability of the method and its robustness are derived by Castagna and Mercurio (2007) in [1].

The following is based on the section *Traders' rule of Thumb* by Wystup in [6].

The traders' rule of thumb is a method of traders to determine the cost of risk managing the volatility risk of exotic options with vanilla options. This cost is then added to the theoretical value in the Black-Scholes model and is called the *overhedge*. We explain the rule and then consider an example of a one-touch.

Delta and vega are the most relevant sensitivity parameters for foreign exchange options maturing within one year. A delta-neutral position can be achieved by trading the spot. Changes in the spot are explicitly allowed in the Black-Scholes model. Therefore, model and practical trading have very good control over spot change risk. The more sensitive part is the vega position. This is not taken care of in the Black-Scholes model. Market participants need to trade other options to obtain a vega-neutral position. However, even a vega-neutral position is subject to changes of spot and volatility. For this reason, the sensitivity parameters *vanna* (change of vega due to change of spot) and *volga* (change of vega due to change of volatility) are of special interest. Vanna is also called *d vega /d spot*, volga is also called *d vega /d vol*. The plots for vanna and volga for a vanilla are displayed in Figures 1 and 2. In this section we outline how the cost of such a vanna- and volga- exposure can be used to obtain prices for options that are closer to the market than their theoretical Black-Scholes value.

1.1 Cost of Vanna and Volga

We fix the rates r_d and r_f, the time to maturity T and the spot x and define

$$
\begin{aligned}
\text{cost of vanna} &\stackrel{\Delta}{=} \text{Exotic Vanna Ratio} \times \text{value of RR}, & (1)\\
\text{cost of volga} &\stackrel{\Delta}{=} \text{Exotic Volga Ratio} \times \text{value of BF}, & (2)\\
\text{Exotic Vanna Ratio} &\stackrel{\Delta}{=} B_{\sigma x}/\text{RR}_{\sigma x}, & (3)\\
\text{Exotic Volga Ratio} &\stackrel{\Delta}{=} B_{\sigma\sigma}/\text{BF}_{\sigma\sigma}, & (4)\\
\text{value of RR} &\stackrel{\Delta}{=} [\text{RR}(\sigma_\Delta) - \text{RR}(\sigma_0)], & (5)\\
\text{value of BF} &\stackrel{\Delta}{=} [\text{BF}(\sigma_\Delta) - \text{BF}(\sigma_0)], & (6)
\end{aligned}
$$

where σ_0 denotes the at-the-money (forward) volatility and σ_Δ denotes the wing volatility at the delta pillar Δ, B denotes the value function of a given exotic option. The values of risk

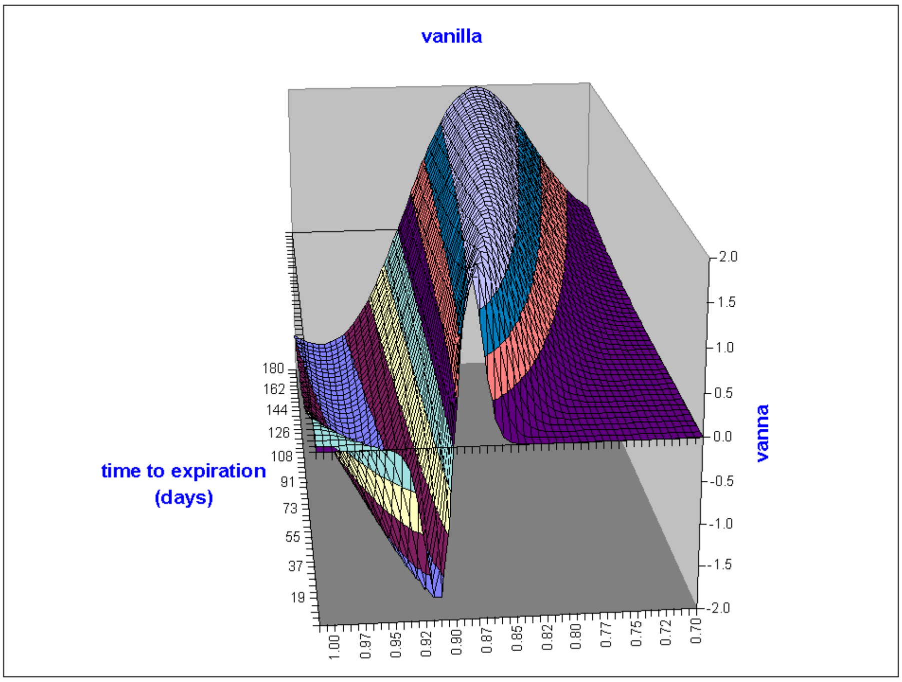


Figure 1: Vanna of a vanilla option as a function of spot and time to expiration, showing the skew symmetry about the at-the-money line

reversals and butterflies are defined by

$$\begin{aligned}\mathsf{RR}(\sigma) &\stackrel{\Delta}{=} \mathsf{call}(x,\Delta,\sigma,r_d,r_f,T) - \mathsf{put}(x,\Delta,\sigma,r_d,r_f,T), &(7)\\ \mathsf{BF}(\sigma) &\stackrel{\Delta}{=} \frac{\mathsf{call}(x,\Delta,\sigma,r_d,r_f,T) + \mathsf{put}(x,\Delta,\sigma,r_d,r_f,T)}{2}\\ &- \frac{\mathsf{call}(x,\Delta_0,\sigma_0,r_d,r_f,T) + \mathsf{put}(x,\Delta_0,\sigma_0,r_d,r_f,T)}{2}, &(8)\end{aligned}$$

where $\mathsf{vanilla}(x,\Delta,\sigma,r_d,r_f,T)$ means $\mathsf{vanilla}(x,K,\sigma,r_d,r_f,T)$ for a strike K chosen to imply $|\mathsf{vanilla}_x(x,K,\sigma,r_d,r_f,T)| = \Delta$ and Δ_0 is the delta that produces the at-the-money strike. To summarize we abbreviate

$$\begin{aligned}\mathsf{c}(\sigma_\Delta^+) &\stackrel{\Delta}{=} \mathsf{call}(x,\Delta^+,\sigma_\Delta^+,r_d,r_f,T), &(9)\\ \mathsf{p}(\sigma_\Delta^-) &\stackrel{\Delta}{=} \mathsf{put}(x,\Delta^-,\sigma_\Delta^-,r_d,r_f,T), &(10)\end{aligned}$$

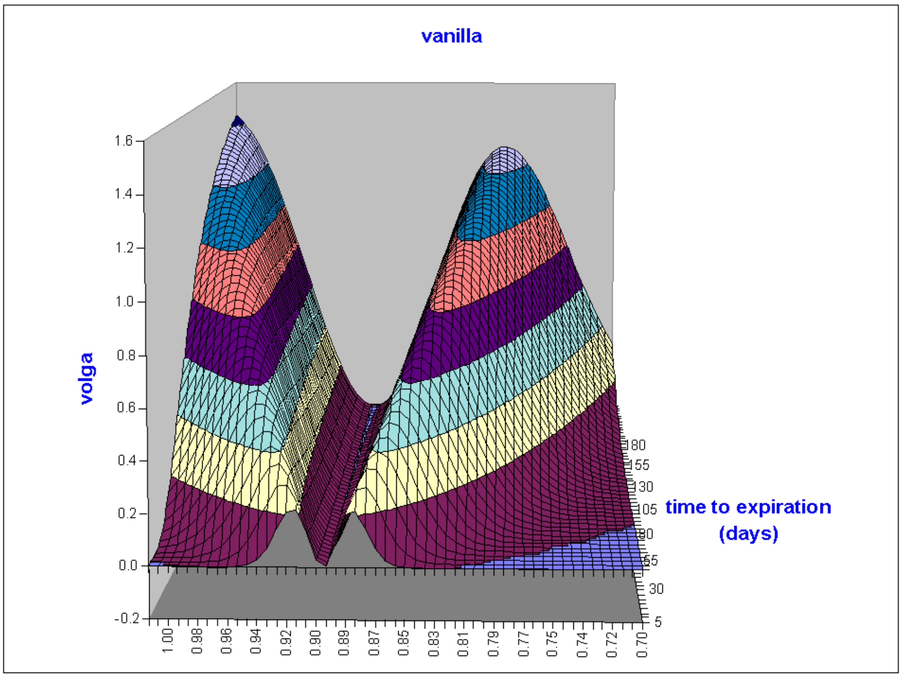


Figure 2: Volga of a vanilla option as a function of spot and time to expiration, showing the symmetry about the at-the-money line

and obtain

$$\text{cost of vanna} = \frac{B_{\sigma x}}{\mathsf{c}_{\sigma x}(\sigma_\Delta^+) - \mathsf{p}_{\sigma x}(\sigma_\Delta^-)} \left[\mathsf{c}(\sigma_\Delta^+) - \mathsf{c}(\sigma_0) - \mathsf{p}(\sigma_\Delta^-) + \mathsf{p}(\sigma_0)\right], \tag{11}$$

$$\text{cost of volga} = \frac{2B_{\sigma\sigma}}{\mathsf{c}_{\sigma\sigma}(\sigma_\Delta^+) + \mathsf{p}_{\sigma\sigma}(\sigma_\Delta^-)} \left[\frac{\mathsf{c}(\sigma_\Delta^+) - \mathsf{c}(\sigma_0) + \mathsf{p}(\sigma_\Delta^-) - \mathsf{p}(\sigma_0)}{2}\right], \tag{12}$$

where we note that volga of the butterfly should actually be

$$\frac{1}{2}\left[\mathsf{c}_{\sigma\sigma}(\sigma_\Delta^+) + \mathsf{p}_{\sigma\sigma}(\sigma_\Delta^-) - \mathsf{c}_{\sigma\sigma}(\sigma_0) - \mathsf{p}_{\sigma\sigma}(\sigma_0)\right], \tag{13}$$

but the last two summands are close to zero. The *vanna-volga adjusted value* of the exotic is then

$$B(\sigma_0) + p \times [\text{cost of vanna} + \text{cost of volga}]. \tag{14}$$

A division by the spot x converts everything into the usual quotation of the price in % of the underlying currency. The cost of vanna and volga is often adjusted by a number $p \in [0, 1]$, which is often taken to be the risk-neutral no-touch probability. The reason is that in the case of options that can knock out, the hedge is not needed anymore once the option has knocked out. The exact choice of p depends on the product to be priced; see Table 2. Taking $p = 1$ as the default valuewould lead to overestimated overhedges for double-no-touch options as pointed out in [2].

The values of risk reversals and butterflies in Equations (11) and (12) can be approximated by a first order expansion as follows. For a risk reversal we take the difference of the call with correct implied volatility and the call with at-the-money volatility minus the difference of the put with correct implied volatility and the put with at-the-money volatility. It is easy to see that this can be well approximated by the vega of the at-the-money vanilla times the risk reversal in terms of volatility. Similarly the cost of the butterfly can be approximated by the vega of the at-the-money volatility times the butterfly in terms of volatility. In formulae this is

$$\begin{aligned} & \mathsf{c}(\sigma_\Delta^+) - \mathsf{c}(\sigma_0) - \mathsf{p}(\sigma_\Delta^-) + \mathsf{p}(\sigma_0) \\ \approx\ & \mathsf{c}_\sigma(\sigma_0)(\sigma_\Delta^+ - \sigma_0) - \mathsf{p}_\sigma(\sigma_0)(\sigma_\Delta^- - \sigma_0) \\ =\ & \sigma_0[\mathsf{p}_\sigma(\sigma_0) - \mathsf{c}_\sigma(\sigma_0)] + \mathsf{c}_\sigma(\sigma_0)[\sigma_\Delta^+ - \sigma_\Delta^-] \\ =\ & \mathsf{c}_\sigma(\sigma_0)\mathsf{RR} \end{aligned} \tag{15}$$

and similarly

$$\begin{aligned} & \frac{\mathsf{c}(\sigma_\Delta^+) - \mathsf{c}(\sigma_0) + \mathsf{p}(\sigma_\Delta^-) - \mathsf{p}(\sigma_0)}{2} \\ \approx\ & \mathsf{c}_\sigma(\sigma_0)\mathsf{BF}. \end{aligned} \tag{16}$$

With these approximations we obtain the formulae

$$\text{cost of vanna} \approx \frac{B_{\sigma x}}{\mathsf{c}_{\sigma x}(\sigma_\Delta^+) - \mathsf{p}_{\sigma x}(\sigma_\Delta^-)}\mathsf{c}_\sigma(\sigma_0)\mathsf{RR}, \tag{17}$$

$$\text{cost of volga} \approx \frac{2B_{\sigma\sigma}}{\mathsf{c}_{\sigma\sigma}(\sigma_\Delta^+) + \mathsf{p}_{\sigma\sigma}(\sigma_\Delta^-)}\mathsf{c}_\sigma(\sigma_0)\mathsf{BF}. \tag{18}$$

1.2 Observations

1. The price supplements are linear in butterflies and risk reversals. In particular, there is no cost of vanna supplement if the risk reversal is zero and no cost of volga supplement if the butterfly is zero.

2. The price supplements are linear in the at-the-money vanilla vega. This means supplements grow with growing volatility change risk of the hedge instruments.

3. The price supplements are linear in vanna and volga of the given exotic option.

4. We have not observed any relevant difference between the exact method and its first order approximation. Since the computation time for the approximation is shorter, we recommend using the approximation.

5. It is not clear up front which target delta to use for the butterflies and risk reversals. We take a delta of 25% merely on the basis of its liquidity.

6. The prices for vanilla options are consistent with the input volatilities as shown in Figures 3 , 4 and 5.

7. The method assumes a zero volga of risk reversals and a zero vanna of butterflies. This way the two sources of risk can be decomposed and hedged with risk reversals and butterflies. However, the assumption is actually not exact. For this reason, the method should be used with a lot of care. It causes traders and financial engineers to keep adding exceptions to the standard method.

1.3 Consistency Check

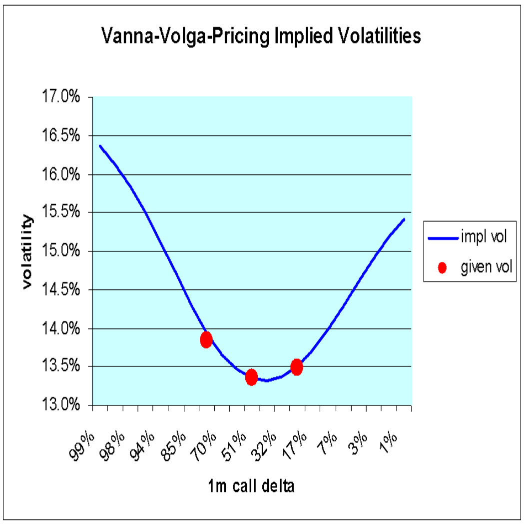


Figure 3: Consistency check of vanna-volga-pricing. Vanilla option smile for a one month maturity EUR/USD call, spot $= 0.9060$, $r_d = 5.07\%$, $r_f = 4.70\%$, $\sigma_0 = 13.35\%$, $\sigma_\Delta^+ = 13.475\%$, $\sigma_\Delta^- = 13.825\%$

A minimum requirement for the vanna-volga pricing to be correct is the consistency of the method with vanilla options. We show in Figure 3, Figure 4 and Figure 5 that the method does in fact yield a typical foreign exchange smile shape and produces the correct input volatilities at-the-money and at the delta pillars. We will now prove the consistency in the following way.

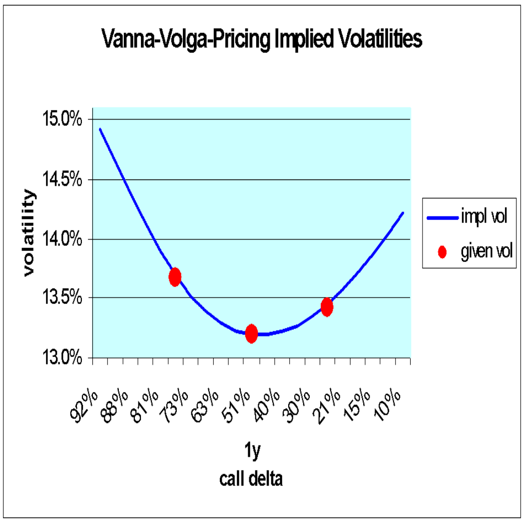


Figure 4: Consistency check of vanna-volga-pricing. Vanilla option smile for a one year maturity EUR/USD call, spot = 0.9060, $r_d = 5.07\%$, $r_f = 4.70\%$, $\sigma_0 = 13.20\%$, $\sigma_\Delta^+ = 13.425\%$, $\sigma_\Delta^- = 13.575\%$

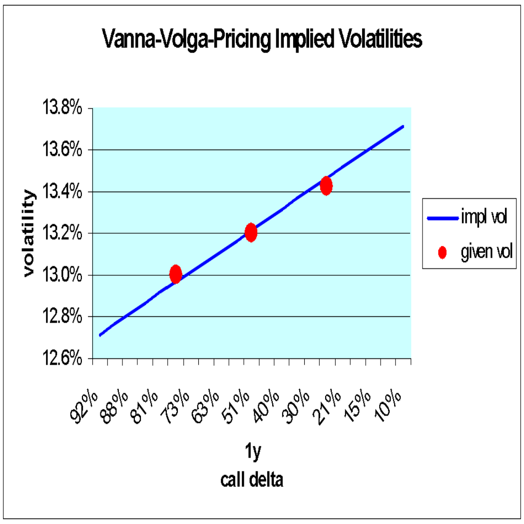


Figure 5: Consistency check of vanna-volga-pricing. Vanilla option smile for a one year maturity EUR/USD call, spot = 0.9060, $r_d = 5.07\%$, $r_f = 4.70\%$, $\sigma_0 = 13.20\%$, $\sigma_\Delta^+ = 13.425\%$, $\sigma_\Delta^- = 13.00\%$

Since the input consists only of three volatilities (at-the-money and two delta pillars), it would be too much to expect that the method produces correct representation of the entire volatility matrix. We can only check if the values for at-the-money and target-Δ puts and calls are reproduced correctly. In order to verify this, we check if the values for an at-the-money call, a risk reversal and a butterfly are priced correctly. Of course, we only expect approximately correct results. Note that the number p is taken to be one, which agrees with the risk-neutral no-touch probability for vanilla options.

For an at-the-money call vanna and volga are approximately zero, whence there are no supplements due to vanna cost or volga cost.

For a target-Δ risk reversal

$$\mathsf{c}(\sigma_\Delta^+) - \mathsf{p}(\sigma_\Delta^-) \tag{19}$$

we obtain

$$\begin{aligned}
\text{cost of vanna} &= \frac{\mathsf{c}_{\sigma x}(\sigma_\Delta^+) - \mathsf{p}_{\sigma x}(\sigma_\Delta^-)}{\mathsf{c}_{\sigma x}(\sigma_\Delta^+) - \mathsf{p}_{\sigma x}(\sigma_\Delta^-)} \left[\mathsf{c}(\sigma_\Delta^+) - \mathsf{c}(\sigma_0) - \mathsf{p}(\sigma_\Delta^-) + \mathsf{p}(\sigma_0)\right] \\
&= \mathsf{c}(\sigma_\Delta^+) - \mathsf{c}(\sigma_0) - \mathsf{p}(\sigma_\Delta^-) + \mathsf{p}(\sigma_0), \qquad (20) \\
\text{cost of volga} &= \frac{2[\mathsf{c}_{\sigma\sigma}(\sigma_\Delta^+) - \mathsf{p}_{\sigma\sigma}(\sigma_\Delta^-)]}{\mathsf{c}_{\sigma\sigma}(\sigma_\Delta^+) + \mathsf{p}_{\sigma\sigma}(\sigma_\Delta^-)} \\
&\quad \left[\frac{\mathsf{c}(\sigma_\Delta^+) - \mathsf{c}(\sigma_0) + \mathsf{p}(\sigma_\Delta^-) - \mathsf{p}(\sigma_0)}{2}\right], \qquad (21)
\end{aligned}$$

and observe that the cost of vanna yields a perfect fit and the cost of volga is small, because in the first fraction we divide the difference of two quantities by the sum of the quantities, which are all of the same order.

For a target-Δ butterfly

$$\frac{\mathsf{c}(\sigma_\Delta^+) + \mathsf{p}(\sigma_\Delta^-)}{2} - \frac{\mathsf{c}(\sigma_0) + \mathsf{p}(\sigma_0)}{2} \tag{22}$$

we analogously obtain a perfect fit for the cost of volga and

$$\begin{aligned}
\text{cost of vanna} &= \frac{\mathsf{c}_{\sigma x}(\sigma_\Delta^+) - \mathsf{p}_{\sigma x}(\sigma_0) - [\mathsf{c}_{\sigma x}(\sigma_0) - \mathsf{p}_{\sigma x}(\sigma_\Delta^-)]}{\mathsf{c}_{\sigma x}(\sigma_\Delta^+) - \mathsf{p}_{\sigma x}(\sigma_0) + [\mathsf{c}_{\sigma x}(\sigma_0) - \mathsf{p}_{\sigma x}(\sigma_\Delta^-)]} \\
&\quad \left[\mathsf{c}(\sigma_\Delta^+) - \mathsf{c}(\sigma_0) - \mathsf{p}(\sigma_\Delta^-) + \mathsf{p}(\sigma_0)\right], \qquad (23)
\end{aligned}$$

which is again small.

The consistency can actually fail for certain parameter scenarios. This is one of the reasons, why the traders' rule of thumb has been criticized repeatedly by a number of traders and researchers.

1.4 Abbreviations for First Generation Exotics

We introduce the abbreviations for first generation exotics listed in Table 1.

KO	knock-out
KI	knock-in
RKO	reverse knock-out
RKI	reverse knock-in
DKO	double knock-out
OT	one-touch
NT	no-touch
DOT	double one-touch
DNT	double no-touch

Table 1: Abbreviations for first generation exotics

1.5 Adjustment Factor

The factor p has to be chosen in a suitable fashion. Since there is no mathematical justification or indication, there is a lot of dispute in the market about this choice. Moreover, the choices may also vary over time. An example for *one of many possible choices* of p is presented in Table 2.

product	p
KO	no-touch probability
RKO	no-touch probability
DKO	no-touch probability
OT	0.9 * no-touch probability - 0.5 * bid-offer-spread *(TV - 33%) / 66%
DNT	0.5

Table 2: Adjustment factors for the overhedge for first generation exotics

For options with strike K, barrier B and type $\phi = 1$ for a call and $\phi = -1$ for a put, we use the following pricing rules which are based on no-arbitrage conditions.

KI is priced via $KI =$ *vanilla* $- KO$.

RKI is priced via $RKI =$ *vanilla* $- RKO$.

RKO is priced via
$$RKO(\phi, K, B) = KO(-\phi, K, B) - KO(-\phi, B, B) + \phi(B - K)NT(B).$$

DOT is priced via DNT.

NT is priced via OT.

1.6 Volatility for Risk Reversals, Butterflies and Theoretical Value

To determine the volatility and the vanna and volga for the risk–reversal and butterfly, the convention is the same as for the building of the smile curve. Hence the 25% delta risk–reversal retrieves the strike for 25% delta call and put with the spot delta and premium included [left-hand-side in *Fenics*] and calculates the vanna and volga of these options using the corresponding volatilities from the smile.
The theoretical value (TV) of the exotics is calculated using the ATM–volatility retrieving it with the same convention that was used to built the smile, i.e. delta–parity with premium included [left-hand-side in *Fenics*].

1.7 Pricing Barrier Options

Ideally one would be in a situation to hedge all barrier contracts with portfolio of vanilla options or simple barrier building blocks. In the Black-Scholes model there are exact rules how to statically hedge many barrier contracts. A state of the art reference is given by Poulsen (2006) in [3]. However, in practice most of these hedges fail, because volatility is not constant. For regular knock-out options one can refine the method to incorporate more information about the global shape of the vega surface through time.

We chose M future points in time as $0 < a_1\% < a_2\% < \ldots < a_M\%$ of the time to expiration. Using the same cost of vanna and volga we calculate the overhedge for the regular knock-out with a reduced time to expiration. The factor for the cost is the no-touch probability to touch the barrier within the remaining times to expiration $1 > 1 - a_1\% > 1 - a_2\% > \ldots > 1 - a_M\%$ of the total time to expiration. Some desks believe that for at-the-money strikes the long time should be weighted higher and for low delta strikes the short time to maturity should be weighted higher. The weighting can be chosen (rather arbitrarily) as

$$w = \tanh[\gamma(|\delta - 50\%| - 25\%)] \tag{24}$$

with a suitable positive γ. For $M = 3$ the total overhedge is given by

$$\mathsf{OH} = \frac{\mathsf{OH}(1 - a_1\%) * w + \mathsf{OH}(1 - a_2\%) + \mathsf{OH}(1 - a_3\%) * (1 - w)}{3}. \tag{25}$$

Which values to use for M, γ and the a_i, whether to apply a weighting and what kind varies for different trading desks.
An additional term can be used for single barrier options to account for glitches in the stop–loss of the barrier. The theoretical value of the barrier option is determined with a barrier that is moved by 4 basis points and 50% of that adjustment is added to the price if it is positive. If it is negative it is omitted altogether. The theoretical foundation for such a method is explained in [4].

1.8 Pricing Double Barrier Options

Double barrier options behave similar to vanilla options for a spot far away from the barrier and more like one-touch options for a spot close to the barrier. Therefore, it appears reasonable to use the traders' rule of thumb for the corresponding regular knock-out to determine the overhedge for a spot closer to the strike and for the corresponding one-touch for a spot closer to the barrier. This adjustment is the intrinsic value of the reverse knock-out times the overhedge of the corresponding one-touch. The border is the arithmetic mean between strike and the in-the-money barrier.

1.9 Pricing Double-No-Touch Options

For double-no-touch options with lower barrier L and higher barrier H at spot S one can use the overhedge

$$\mathsf{OH} = \max\{\text{Vanna-Volga-OH}; \delta(S - L) - \mathsf{TV} - 0.5\%; \delta(H - S) - \mathsf{TV} - 0.5\%\}, \tag{26}$$

where δ denotes the delta of the double-no-touch.

1.10 Pricing European Style Options

1.10.1 Digital Options

Digital options are priced using the overhedge of the call– or put–spread with the corresponding volatilities.

1.10.2 European Barrier Options

European barrier options (EKO) are priced using the prices of European and digital options and the relationship

$$\mathsf{EKO}(\phi, K, B) = \mathsf{vanilla}(\phi, K) - \mathsf{vanilla}(\phi, B) - \mathsf{digital}(B)\phi(B-K). \tag{27}$$

1.11 No-Touch Probability

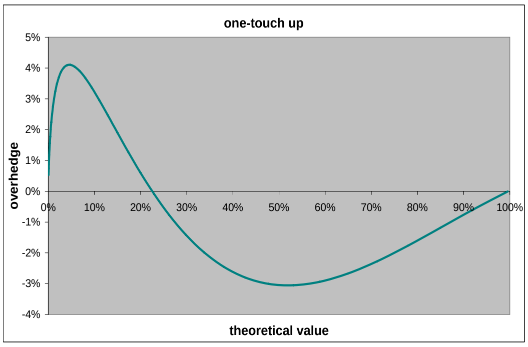
The no-touch probability is obviously equal to the non-discounted value of the corresponding no–touch option paying at maturity (under the risk neutral measure). Note that the price of the one-touch is calculated using an iteration for the touch probability. This means that the price of the one-touch used to compute the no-touch probability is itself based on the the traders' rule of thumb. This is an iterative process which requires an abortion criterion. One can use a standard approach that ends either after 100 iterations or as soon as the difference of two successive iteration results is less than 10^{-6}. However, the method is so crude that it actually does not make much sense to use such precision at just this point. So in order to speed up the computation we suggest to omit this procedure and take zero iterations, which is the TV of the no–touch.

1.12 The Cost of Trading and its Implication on the Market Price of One-touch Options

Now let us take a look at an example of the traders' rule of thumb in its simple version. We consider one-touch options, which hardly ever trade at TV. The tradable price is the sum of the TV and the overhedge. Typical examples are shown in Figure 6, one for an upper touch level in EUR-USD, one for a lower touch level.

Clearly there is no overhedge for one-touch options with a TV of 0% or 100%, but it is worth noting that low-TV one-touch options can be twice as expensive as their TV, sometimes even more. *SuperDerivatives* [1] has become one of the standard references of pricing exotic FX options up to the market. The overhedge arises from the cost of risk managing the one-touch. In the Black-Scholes model, the only source of risk is the underlying exchange rate, whereas the volatility and interest rates are assumed constant. However, volatility and rates are themselves changing, whence the trader of options is exposed to instable vega and rho (change of the value with respect to volatility and rates). For short dated options, the interest rate risk is negligible compared to the volatility risk as shown in Figure 7. Hence the overhedge of a one-touch is a reflection of a trader's cost occurring due to the risk management of his vega exposure.

[1] http://www.superderivatives.com



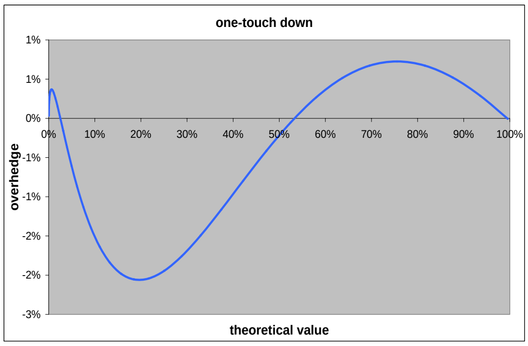


Figure 6: Overhedge of a one-touch in EUR-USD for both an upper touch level and a lower touch level, based on the traders' rule of thumb

1.13 Example

We consider a one-year one-touch in USD/JPY with payoff in USD. As market parameters we assume a spot of 117.00 JPY per USD, JPY interest rate 0.10%, USD interest rate 2.10%, volatility 8.80%, 25-delta risk reversal -0.45%[2] , 25-delta butterfly 0.37%[3].

The touch level is 127.00, and the TV is at 28.8%. If we now only hedge the vega exposure, then we need to consider two main risk factors, namely

1. the change of vega as the spot changes, often called vanna,
2. the change of vega as the volatility changes, often called volga or volgamma or vomma.

To hedge this exposure we treat the two effects separately. The vanna of the one-touch is 0.16%, the vanna of the risk reversal is 0.04%. So we need to buy 4 (=0.16/0.04) risk reversals, and for each of them we need to pay 0.14% of the USD amount, which causes an overhedge of -0.6%. The volga of the one-touch is -0.53%, the volga of the butterfly is 0.03%. So we need to sell 18 (=-0.53/0.03) butterflies, each of which pays us 0.23% of the USD amount, which causes an overhedge of -4.1%. Therefore, the overhedge is -4.7%. However, we will get to the touch level with a risk-neutral probability of 28.8%, in which case we would have to pay to unwind the hedge. Therefore the total overhedge is -71.2%*4.7%=-3.4%. This leads to a mid market price of 25.4%. Bid and offer could be 24.25% – 36.75%. There are

[2]This means that a 25-delta USD call is 0.45% cheaper than a 25-delta USD put in terms of implied volatility.

[3]This means that a 25-delta USD call and 25-delta USD put is on average 0.37% more expensive than an at-the-money option in terms of volatility

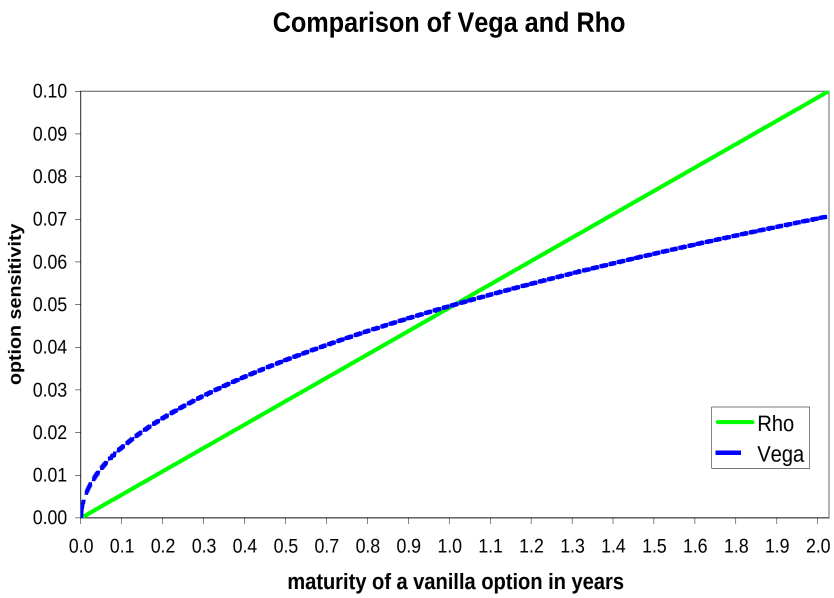


Figure 7: Comparison of interest rate and volatility risk for a vanilla option. The volatility risk behaves like a square root function, whereas the interest rate risk is close to linear. Therefore, short-dated FX options have higher volatility risk than interest rate risk.

different beliefs among market participants about the unwinding cost. Other observed prices for one-touch options can be due to different existing vega profiles of the trader's portfolio, a marketing campaign, a hidden additional sales margin or even the overall condition of the trader in charge.

1.14 Further Applications

The method illustrated above shows how important the current smile of the vanilla options market is for the pricing of simple exotics. Similar types of approaches are commonly used to price other exotic options. For long-dated options the interest rate risk will take over the lead in comparison to short dated options where the volatility risk is dominant.

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