Volatility as Investment - Crash Protection with Calendar Spreads of Variance Swaps*

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Abstract

Nowadays, volatility is not only a risk measure but can be also considered an individual asset class. Variance swaps, one of the main investment vehicles, can obtain pure exposure on realized volatility. In normal market phases, implied volatility is often higher than the realized volatility will turn out to be.

In this paper, we show a volatility investment strategy which can benefit from both negative risk premium and correlation of variance swaps to the underlying stock index. The empirical evidence demonstrate a significant diversification effect during the financial crisis by adding this strategy to the equity portfolio. The back testing analysis includes the last ten years of history of the S&P500 and the EUROSTOXX50.

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1 Introduction

Volatility is an important variable of the financial market. The accurate measurement of volatility is important not only for investment but also as an integral part of risk management. Generally, there are two methods used to predict volatility: The first one entails the application of time series methods in order to make an accurate prediction based on historical data. Examples of this method are ARCH/GARCH models. The second method is based on implied volatility from option prices, which reflects the investor's view on future volatility. Historically, volatility used to be traded through option strategies such as butterflies or strangles. Using such a strategy exposes investors to the risks of implied volatility. However, holding option strategies also exposes investors to risks associated to, for example, interest rates and the underlying (referred to by the greek letters Delta, Gamma, Theta, etc.). Nowadays, products such as variance swaps and volatility swaps and their variantes allow for a pure volatility investment. Due to the increasingly important role of volatility, the first volatility index (Ticker:VXO) was launched by the Chicago Board Options Exchange (CBOE) in 1993. Since being revised to VIX in 2004, the index provides a quote of the fair strike for a variance swap with a 30 day term for the S&P 500 index. In 2004, the VIX future was launched, followed by VIX options in 2006.

Both the VIX level and implied volatilities from options are often used as means to predict future fluctuations in the market. It is often said to be "the investor fear gauge.[1]", as the VIX index is often very quickly pushed up by investors' fear in the market. The past years in particular are marked by an increase in both trading volume and an outstanding amount of VIX future, as can be seen in Figure 1. Not limited to the US market, individual investors can now easily trade volatility for most major equity indices (e.g. EuroStoxx, Nikkei, S&P 500) all over the world and at different term structures either by trading options or futures on indices similar to the VIX. The sharp increase in trading volume of VIX futures observed during the sub prime crisis reflects investors' intention to diversify in order to protect their investment.

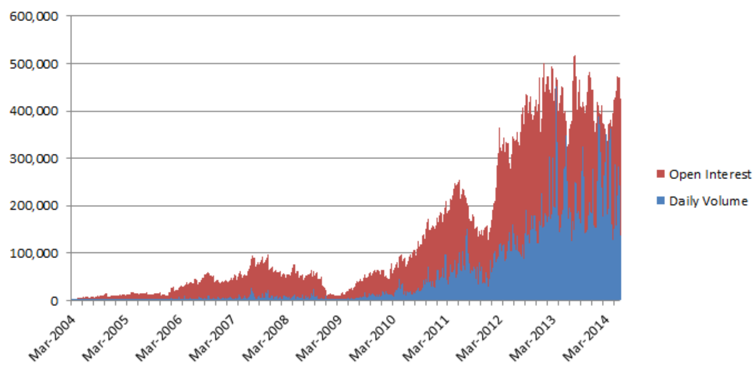

Figure 1: The Trading Volume of VIX Future. Source: `http://cfe.cboe.com/Products/historicalVIX.aspx`

[1]See Whaley (2000);

There are further incentives of trading volatility. Unlike other assets, volatility generally shows a mean-reverting characteristic; it can grow neither to arbitrary high or low levels, but tends towards a medium level in the long run. It is well documented that volatility is negatively correlated with the stock index[2]. This negative correlation is explained by Black (1976) as the leverage hypothesis, where a decline in the equity level increases the leverage of the firm (market) and hence the risk to the stock (index). By studying the S&P 100 / S&P 500 indices and their volatility indices, Whaley (2000), Giot (2005) and Hibbert et al. (2008) point out the asymmetry of this negative relationship. While an increase in the VIX is marked by a drop in the stock market, a fall in the VIX is marked by only a small increase in stock market. In Figure 2, we calculate the rolling correlation between the VIX and the S&P 500 index, noting that the correlation remains highly negative (-80%) during the crisis. Dash and Moran (2005) illustrates that implied volatility could provide a robust diversification method to reduce the investment risk[3].

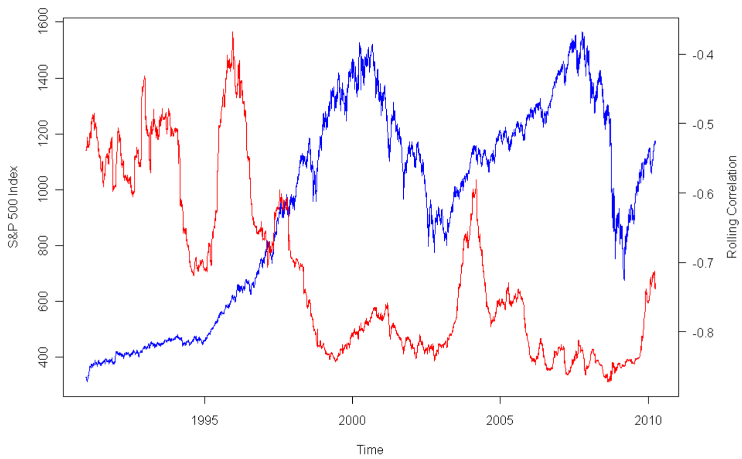

Figure 2: Correlation between S & P 500 Index and VIX

With the growing trading activity of volatility products increases, asset managers also use such products to optimize their investment portfolios. During normal market phases (such as time between 2004-2007 see Figure 3, implied short term volatility is often higher than realized volatility. This negative risk premium for implied variance has been studied in several articles[4]. Carr and Wu (2009) argue that an increase in market volatility is an unfavorable shock to investors and that they would thus be willing to pay a protection premium. Our estimates suggest that the average negative variance premium is around 3.5% in volatility for 30 day variance swap contracts, and 2.9% in volatility for 1 year variance swap contracts. Hafner and Wallmeier (2007) and Grant et al. (2007) propose

[2]See such as Daigler and Rossi (2006),Whaley (2000) and Giot (2005)

[3]The correlation among stocks rose towards one during the financial crisis and the traditional diversification effect was low.

[4]see for example Carr and Wu (2009) and Hafner and Wallmeier (2007).

to add an exclusive variance swap selling strategy to a portfolio in order to earn this premium. However, a strategy based exclusively on selling does not make use of the negative correlation between the variance swaps and the underlying asset, thus rendering the diversification effect trivial. Szado (2009) points out that a long volatility strategy would significantly help investors to diversify and protect their portfolio during the financial crisis. However, he does admit that this would be an expensive means of protection as it causes negative returns during normal market phases.

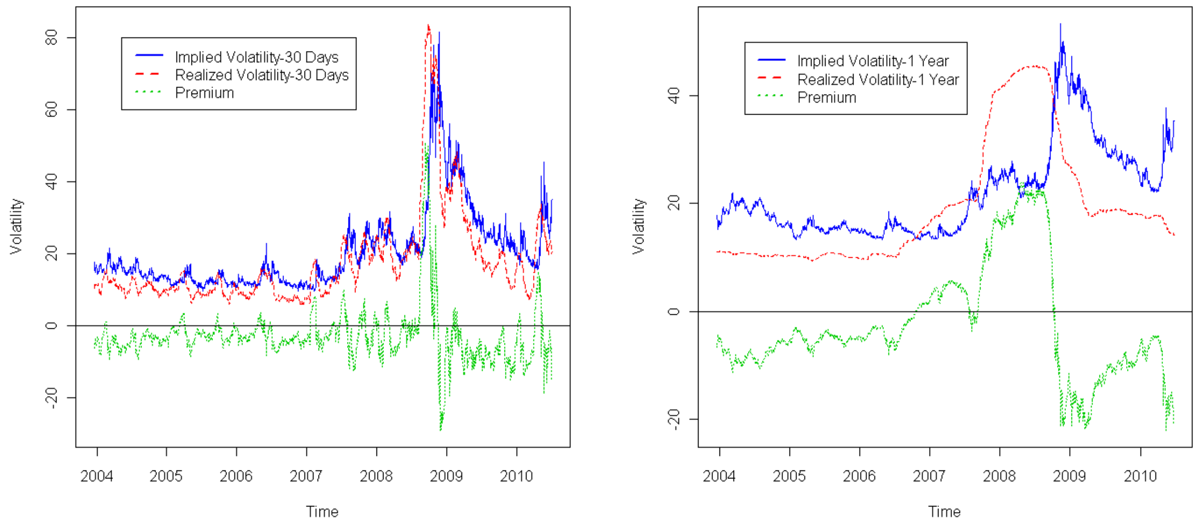

Figure 3: Negative Premium of Variance Swaps

In this study, complementing existing research, we introduce a trading strategy of implied volatility that provides diversification at almost zero cost. We use the idea of calendar spreads from option trading and apply it to variance swaps. A calendar spread of variance swaps combines selling short term and buying long term variance swaps. This strategy can avoid the high costs encountered in single buying strategies and the low diversification effects in simple selling strategies. The key idea here lies in buying the variance swaps with long maturities at low frequencies while selling short term swaps at a higher frequency. At each trading point, we weigh the long term contract with a much higher notional than the short term ones in order to maintain a net zero vega. In the empirical study part, we use original broker quotes for 1, 3, 6 and 12 month variance swaps between 2004 and 2011. We assume our original investment portfolio to consist of 70% S&P 500 and 30% fixed income investment. We apply a calendar spread of buying 1 year and selling 1 month variance swaps to diversify the portfolio. Our estimates show that the addition of this strategy raises the mean return of the original portfolio by around 250%, while reducing the realized portfolio volatility to 50% of the original portfolio's volatility.

The remaining paper is organized as follows: Section 2 briefly introduces the valuation model of variance swaps and Section 3 analyzes the performance of trading single variance swaps. In Section 4, the calendar spread settings are introduced and we present the performance of a predefined case. Further, we expand the strategy to different weights and illustrate more general cases of performance. We will show the robustness of uing a calendar spread by shifting the trading dates in Section 5. We conclude in Section 6.

2 Empirical Study Setup

Variance swaps started to be traded in the late 1980s as an over-the-counter (OTC) derivative paying the difference between the realized variance σ_R^2 and the predefined implied variance or strike σ_K^2. Usually the strike σ_K^2 is determined such that the initial value of the swap is zero. The variance swap can be replicated by trading a series of out-of-the-money (OTM) call and put options. The term of a variance swap contract is taken to be 1 month (30 days) and 1 year (365 days) in our study. The fair strike σ_K is the implied volatility, as determined at the beginning of the contract. For simplicity, if the end of a variance swap is not a business day, the settlement of the variance swap contract is adjusted to be the last business day over the duration period of the study. Generally, variance swap contracts are measured in dollar notional (Notional). In our empirical study, we will use fix leg payments (or "DollarAmount") to measure the weights of the variance swaps. The relationship between notional and dollar amount is given by

$$\text{DollarAmount} = \text{Notional} \cdot \sigma_K^2. \tag{1}$$

We denote Vega exposure of variance swaps as Vega_t. It calculates as $\text{Vega}_t = 2 \cdot \text{Notional} \cdot \sigma_K$. The vega exposure will be positive if we buy implied volatility and negative if we sell. The contract ends with a cash payment at maturity, determined by the formula

$$\text{Payoff} = \text{Notional} \cdot (\sigma_R^2 - \sigma_K^2). \tag{2}$$

Here, the realized volatility σ_R is calculated by

$$\sigma_R = \sqrt{\frac{252}{N} \sum_{i=1}^{N} (x_i)^2} \cdot 100. \tag{3}$$

The return is a daily log return $x_i = \ln \frac{S_i}{S_{i-1}}$ and N is the number of business days within the contract. The mark-to-market value of a variance swap at any time $t \in (0, T)$ is the weighted sum of realized variance and implied variance,

$$\text{Price}_t = \text{Notional} \times \text{PV}_t(T) \left[\frac{t}{T} (\sigma_R(0,t))^2 + \frac{T-t}{T} (\sigma_{im}(t,T))^2 - \sigma_K^2 \right] \tag{4}$$

Here, the $PV_t(T)$ is the present value at time t of receiving 1\$ at maturity T. And $\sigma_R(0,t)$ is the realized volatility until time t and $\sigma_{im}(t,T)$ is the implied volatility from time t to maturity. In our study, the $\sigma_{im}(t,T)$ is determined by linear interpolation. The inputs are implied volatilities for term structures with maturity of 1 month, 3 month, 6 month and 1 year. We use the 1 month implied volatility for any implied volatility that is shorter than 30 days and apply a linear interpolation to calculate $\sigma_{implied}(t,T)$ in other cases as

$$\sigma_{implied}(t,T) = impVol_{t_1} + (T - t - t_1) \cdot \frac{impVol_{t_2} - impVol_{t_1}}{t_2 - t_1}, \tag{5}$$

where t_1, t_2 is the time interval that $T - t$ belongs to and $impVol_{t_1}$, $impVol_{t_2}$ are the corresponding implied volatilities. The sensitivity of a variance swap to implied volatility decreases linearly with time as a direct consequence of mark-to-market additivity and is given by

$$\text{Vega} = \frac{\delta \text{Price}_t}{\delta \sigma_{implied}} = \text{Notional} \cdot (2 \cdot \sigma_{implied}) \cdot \frac{T-t}{T}. \tag{6}$$

Here, the $\sigma_{implied}$ is the implied volatility until contract maturity, t is the valuation time and T is the maturity.

3 Single Variance Swap Strategy

We assume our underlying portfolio to be a mix of 70% of S&P 500 index and 30% fixed income investment with an initial value of $1 Million. In the first part of empirical study we want to demonstrate the diversification effect by adding a single variance swap to that 70-30 portfolio. We consider the time interval from January 2004 to June 2011[5], which covers the sub-prime crisis. Further, we use short term (1 month) or long term (12 month) variance swaps. We focus on the daily log change of the stock index and implied volatility. We present basic statistics concerning their features in Table 1, and find that both indices show non-normal characteisticss due to their high kurtosis. We should expect an extreme fat tail in the distribution of implied volatility.

	Mean Return	Volatility	Skew	Kurtosis
VarSwap 1M	0.00% (-0.46% p.a.)	6.59% (104.68% p.a.)	0.59	4.03
VarSwap 1Y	0.02% (4.37% p.a.)	2.43% (38.61% p.a.)	0.53	4.38

Table 1: Summary statistics

In total, there are four scenarios in which we add a single variance swap to a portfolio - namely by selling or buying 1 month/1 year variance swaps. In our study, we assume no transaction costs and the portfolio to be rebalanced at a weekly frequency. From Equation (5), we see that the vega exposure of a variance swap decays linearly with time. In order to maintain a more stable vega exposure, we need to trade a variance swap at a frequency that allows the contracts to overlap each other. Considering the possible transaction costs in practice, we set the trading frequency to be approximately $\frac{1}{4}$ of the

[5]In this time series we only use the business days on which all quotes are available.

tenor. We then have four contracts overlapping each other at any valuation point. So for a 1 month variance swap, we assume it to be traded weekly [6]. The trading frequency for a 1 year contract is quarterly [7]. Another important issue is how to manage the notional of the variance swap. If we measure the weights by notional, the vega exposure will have a positive relationship with the prevailing implied volatility. This in turn would cause more vega risk in turbulent markets than at any other time. In general, a high risk exposure during the financial crisis is not a preferable situation for the investor. Consequently, we propose to use a dollar amount (Equation 1) to manage the notional. When we fix the total dollar amount, we will trade much more notional of variance swap during normal market phases than during turbulence. This dynamic notional management is in line with most investors' preference. Considering the portfolio value of 1 million, we set the dollar amount to be 7500$ for 1 month variance swap at each trading point and $50,000$$ for 1 year variance swap. In order to measure the portfolio performance as well as the potential risks, we use three risk adjusted measures: the Sharpe ratio, Kappa 1 (known as Sortino Ratio) and Kappa 2 (known as Omega Ratio). The results of different portfolios are presented in Table 2 and the cumulative returns are shown in Figure 5.

	Mean Return	Volatility	Sharpe Ratio	Kappa 1	Kappa 2
70/30 Strategy	3.72%	14.70%	0.1476	14.03	5.55
Selling VarSwap 1M	9.49%	17.98%	0.4417	29.17	10.74
Buying VarSwap 1M	-6.65%	12.26%	-0.6686	-25.4765	-14.2876
Selling VarSwap 1Y	4.18%	45.35%	0.0579	7.8531	1.9065
Buying VarSwap 1Y	3.25%	10.24%	0.1656	17.7183	8.4176

Table 2: Single Variance Swap Strategy

With respect to the selling strategy, we realized similar results as Hafner and Wallmeier (2007) and Grant et al. (2007).

The premium we earn from the selling strategy significantly increases the portfolio's mean return. Through the weekly selling of a 1 month variance swap, the portfolio's mean return rises by more than 250% while the volatility only increases by less than 25%. Therefore, we observe much better risk-adjusted performance measures for the portfolio. However, when we look at the cumulative return in Figure 5 between 2008 and 2009, rather than provide protection during the crisis, selling variance swaps further pushes down the portfolio's return. This does not favor investors looking to diversify with volatility.

When it comes to the buying strategy, the volatility of the new portfolio is significantly lower than the original one. During the financial crisis, the variance swap provides protection to the investor and we can not find a significant drop in the total portfolio returns in the buying strategy. However, a high risk premium paid to protect the market from

[6] We assume that a variance swap is traded on each Monday. If Monday is not a business day then we move to the next possible business day.

[7] We assume that a variance swap is traded on the first day of the months January, April, July and October. If that day is not a business day then we move to the next possible business day.

crashing sharply results in a reduction of mean return from the investment. The premiums of buying variance swaps are even higher than the total returns from the index. In the case of buying a 1 month contract we even have a highly negative return at -6.65% per year.

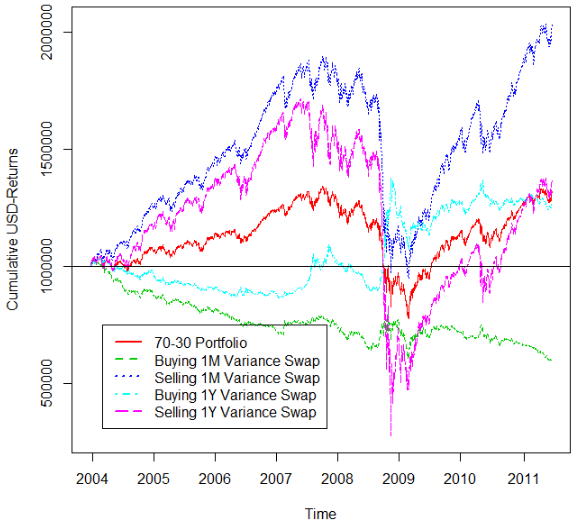

Figure 4: Single Variance Swap Strategy

4 Calendar Spread Strategy

In the last section, we demonstrated the performance achieved when adding single variance swaps to a traditional portfolio. The shortcomings of these strategies are obvious. The selling strategy does not benefit from the diversification and buying one is too expensive to be carried out in a normal market phase. We estimated that the average negative premiums of 1 month and 1 year variance swaps are -51.65 and -46.06 per notional. We can consider the expected marginal cost of buying 1 notional variance swap to be approximately 50\$. If we supposedly want to offer crash protection to the portfolio incase of an increase in volatility, we need to retain a positive vega in our variance swap strategy. Therefore, a long variance swap would be our only choice. This would be expensive and we want to reduce the risk premium. It is not difficult to find that we can sell the variance swaps at the same time to earn a similar amount of risk premium as compensation. Further, the correlation between the movement of 1 month and 1 year implied volatility is strong at 84.56%. We could expect a similar implied volatility curve movement from this high correlation. In practice, a derivatives trader uses a calendar spread to benefit from this parallel volatility curve movement. In the simplest case, we buy a 1 year variance swap and sell a 1 month contract with 1 notional each day. The expected risk premium to be paid on the 1 year swap could be compensated by selling the 1 month swap. Thus, by using a calendar spread structure, we provide a strategy with almost zero expected risk premium. After 1 month the sold contract will expire and the bought 1 year variance swap could provide protection to the portfolio for the remaining 11 months. Within the first month, we hold both, the short and the long contract. From Table 1, we know that the volatility of the 1 month implied volatility is about 2.5 times of the 1 year implied one. Therefore, if we want to offset the returns of a variance swap calendar spread, we need to keep the vega exposure for the long one to be 2.5 times higher than for the short one. It is not very difficult to solve this problem. We can expand the idea above to buying a 1 year variance swap today and selling a 1 month contract in the second month and repeat this for the third month. We combine three simple strategies and now buy 3 times the notional of 1 year variance swaps at the beginning while selling a 1 notional swap on the first day of every month for the next three months. We can keep the vega exposure for the long position to be 2.5 times or more than for the short position. Finally, the expected premium of the long and shot variance swaps offset each other and within the first three months, the return on the two contracts would be well diversified and we get full protection for the remaining 9 months. We can repeat this strategy every quarter and the overlapping strategy would give reasonable protection to the portfolio. So far, we have constructed a calendar spread to provide crash protection to the stock portfolio at almost no cost. The key idea of the calendar spread could be summarized as,

1. We need to have two variance swaps at different maturities;
2. A long position is held for variance swaps with longer maturity and short for the other one;
3. At each trading point, we trade more notional in the long term swaps than in the short term one;
4. We need to sell variance swaps at a higher frequency to match the total notional of the long position;

We will now demonstrate a more general case. Coming back to the example of the single strategy in the last section. We buy a $50,000\$$ 1 year swap quarterly and sell a $7,500\$$ 1 month variance swap contract on a weekly basis. Theoretically, we have four contracts overlapping each other. By estimation, we hold $200,000\$$ in long variance swaps while holding $30,000\$$ in short variance swaps at any time. The vega difference between the long and short strategy is shown as the blue line in Figure 5. If we now combine these two single strategies, the resulting strategy matches the key idea of a calendar spread for variance swaps. In each calender year, we could estimate to sell $360,000\$$ of variance swaps which is greater than the $200,000\$$ buying amount. We are actually earning premium in this situation.

We show the results of the calendar spread variance swap strategy[8]. Figure 5 shows the

[8] As there is no initial investment for the variance swap, we add 1 million dollar cash to the strategy to calculate the performance of this calendar spread.

cumulative return of this strategy and Table 3 presents a summary of the statistics.
In Figure 5, we show cumulative returns of the exclusive short strategy (blue line), long strategy (green line) and calendar spread (red line). Prior to 2008, unlike the portfolio value of the buy strategy which falls below the initial value, our calendar spread actually generates positive returns. When the financial crisis hits in 2008, the calendar spread jumps upward while the value of the short strategy drops heavily. The calendar spread reaches the highest risk adjusted performance in Kappa 1 and Kappa 2. Unlike the negative mean return of the 1 year strategy, it has a significant positive return of 5.43% in our case, which is only about 1% lower than the sell strategy. Finally, we estimate that the correlation between calendar spread and our underlying portfolio is around −79.70%. Recognizing the advantages of the combined calendar spread, we now want to add it to the portfolio to analyze the extent to which the performance can be improved. We present the cumulative returns of the combined portfolio in Figure 6 and summarizing statistics in Table 4. By adding the calendar spread to the 70-30 portfolio, we successfully increase the return from 3.72% to 9.19% while reducing the volatility to 9.63% by almost 5%. We get a Sharpe ratio which is 5 times greater than in the original portfolio, and 2-3 times greater in Kappa 1 and Kappa 2 measures. As we can see from Figure 6, during 2008 the down side drop is significantly reduced by the variance swap. In the density plot shown in Figure 7, we can clearly see that the combined strategy is skewed to the right and that the down side tail distribution is rather small. The density has shifted to the right from the original distribution significantly. In Table 5, we show some risk measures at the extreme condition. By adding a calendar spread, the minimum return and high quantile VaR are significantly reduced. We have thus successfully used two variance swaps with different maturities to construct a calendar spread strategy which correlates highly negatively with the stock index while no or even positive premium is earned.

Strategy	Selling 1-Month	Buying 1-Year	Combined
Mean	6,59%	-1,70%	5,43%
Volatility	5,08%	21,61%	10,28%
Sharpe Ratio	1,2975	-0,0787	0,5284
Kappa 1	29.17	17.7183	49.0037
Kappa 2	10.74	8.4176	20.7640

Table 3: Buying 1 Year, Selling 1 Month variance swap strategy

Taking this idea further, we will weigh a 1Y and a 1M variance swaps differently using the calendar spread strategy. As we know, the portfolio value changes with market fluctuation. A fixed notional would thus not be a good choice with regards to portfolio management. We adjust the notional when buying a 1Y contract to be a percentage of the total portfolio value. As we sell a 1M contract at a smaller amount but at a higher frequency each time, we weigh the dollar amount sold as a proportion of the 1 year variance swap notional. We set the minimum amount of buying a 1Y variance swap to be

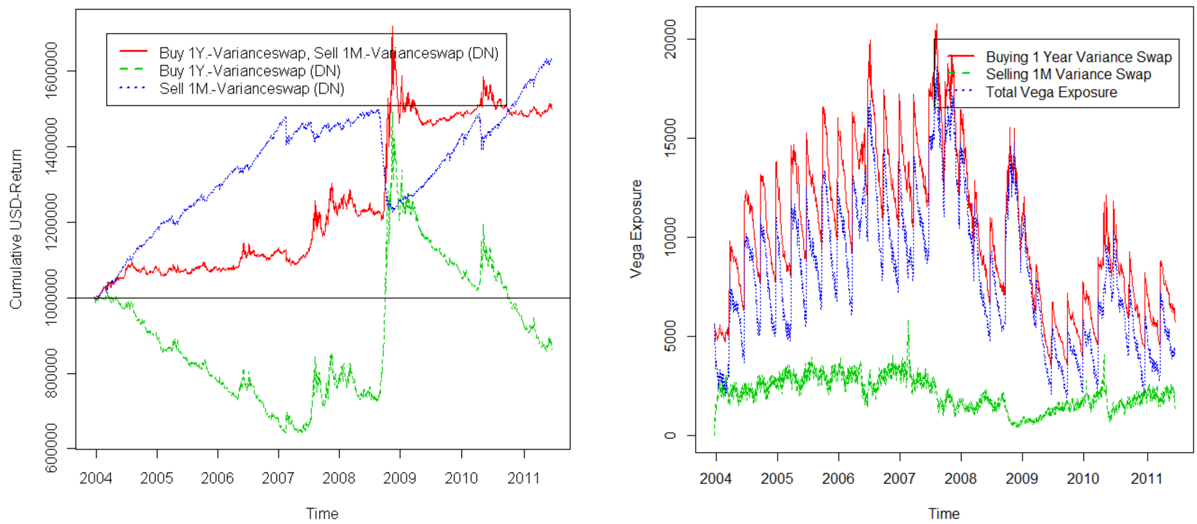

Figure 5: calendar Variance Swap Strategy

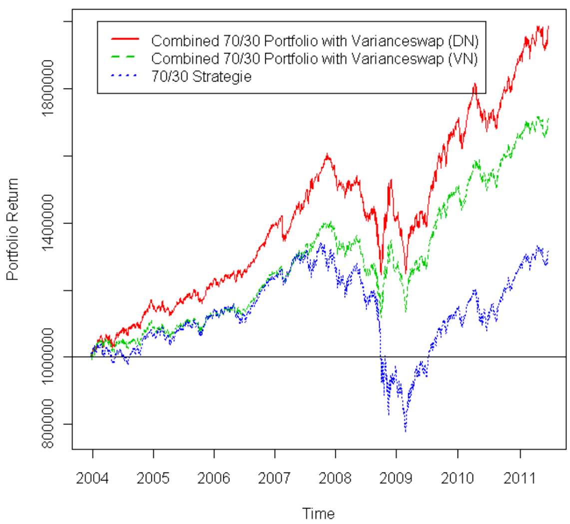

Figure 6: Cumulative Return of Buying 1Year/Selling 1M variance swap

	S&P 500	JPM	70/30 Strat.	Combined (DN)
Mean Return	$2,52\%$	$5,29\%$	$3,72\%$	9,19%
Volatility	$21,40\%$	$4,07\%$	$14,70\%$	9,63%
Sharpe Ratio	0,0441	0,9178	0,1475	0,7924
Kappa 1			14.03	49.0037
Kappa 2			5.55	20.7640

Table 4: Statistic Character of Portfolios

	Min	VaR 99%	VaR 99.9%
70/30 Strat.	-6.54%	-6.21%	-2.90%
Combined	-5.36%	-3.58%	-1.78%

Table 5: Risk measures of Portfolio

1% of the total portfolio value and cap the maximum amount at 10%. The 1M variance swap would have a weight of 5% compared to 10% of the 1Y notional.

We present all results in Figure 8. If we fix the weights of the 1M notional, we see that the Sharpe ratio is a concave function of the 1Y notional. If we only buy a small amount of 1Y variance swaps, it can not provide enough protection during the crisis. However, if the weight a 1Y contract too much, the portfolio would not peform well due to the high premium paid when buying variance swaps. It is thus very important to determine suitable weights for the 1Y variance swap. The portfolio volatility is a convex function of the weights of a 1Y contract. The diversification effect is most significant when buying 3%-4% 1Y variance swaps. In summary, weighting 1Y variance swaps at 3%-4% of the total portfolio value would be a good choice, as this strategy provides both a high Sharpe ratio and the desired diversification effect.

If we fix the amount of long 1Y contracts and sell a larger number of 1M contracts, this would always give us a positive marginal change of the portfolio Sharpe ratio. However, the speed at which it increases would decrease if we weigh them at more than 20% of the long amount. The volatility of the portfolio does not change significantly when we raise the weights of the sold notional. We should observe the effect on the Sharpe ratio. As we stated before, selling single variance swaps would increase the Sharpe ratio but aggregate the loss in the financial crisis. This is why it is important for us to find the right ratio. The figure shows that we can expect a 1Y notional in the range of 15%-20% to give us the best portfolio performance.

5 Robustness of the Strategy

We have seen the significant performance improvement achieved through trading a calendar spread variance swap strategy. However, we assume that the trading occurrs on some specific date. One would argue that the strategy might not work well if we shift

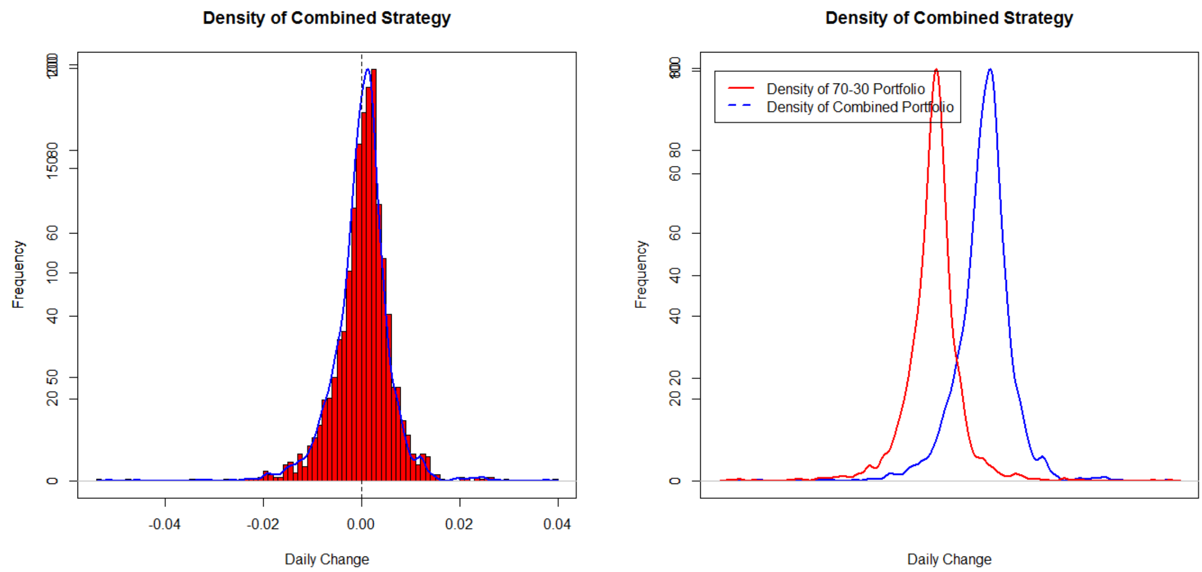

Figure 7: Kernel Density of Combined Portfolio

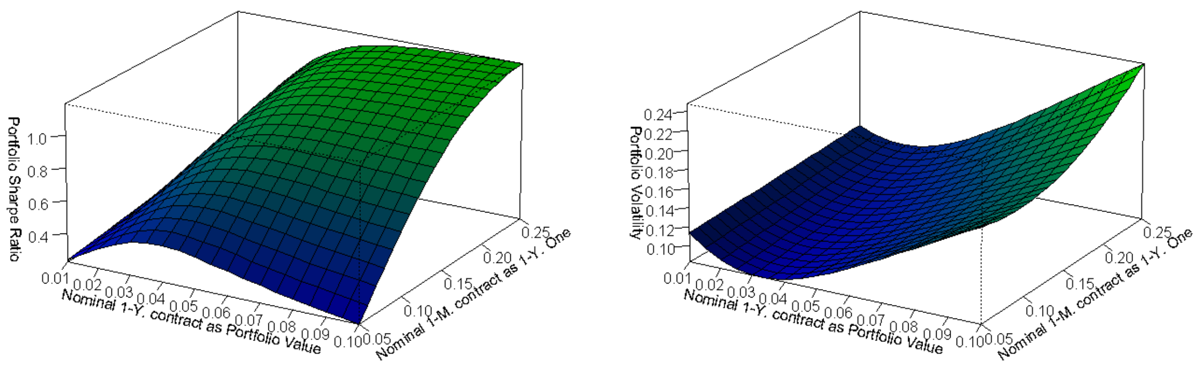

Figure 8: Surface of Sharpe Ratio and Volatility at Different Weight scheme

the trading date. We now want to show the robustness of this calendar spread strategy in the face of a change in dates. We maintain the same trading frequency, but choose a different starting date. For the single 1M strategy, we will show the results if we trade from every Monday to Friday [9] And for selling the 1Y strategy, we shift the starting date by five business days (Weekly). All the results are presented in Table 6 and Table 7. For the selling 1M strategy, we always reach a Sharpe ratio greater than 0.8, in most cases, a Sharpe ratio greater than 1. Similarly, the Sharpe ratio of selling 1Y contracts is always positive. We consider the difference between different Sharpe ratios within an acceptable range. We get similar results when shifting trading days of variance swaps on the EuroStoxx index. The variance swap maintains its own characteristic throughout time.

Day	0	1	2	3	4
Mean Return p.a.	4.78%	4.34%	5.05%	5.46%	5.69%
Sharpe Ratio p.a.	1.0154	0.8868	1.1158	1.2181	1.2420
Difference in %	0.00%	-19.78%	0.94%	10.20%	12.36%

Here, 0-4 corresponds to Monday-Friday;

Table 6: Trading Day Shift of 1M Strategy

Week	0	1	2	3	4	5
Mean Return p.a.	2.15%	2.35%	2.35%	2.08%	2.05%	1.86%
Sharpe Ratio p.a.	0.1069	0.1269	0.1324	0.1078	0.1090	0.0921
Difference in %	0.00%	18.78%	23.89%	0.86%	1.99%	-13.80%
Week	6	7	8	9	10	11
Mean Return (USD)	1.70%	1.75%	2.02%	1.57%	1.80%	1.92%
Sharpe Ratio p.a.	0.0837	0.0868	0.0999	0.0720	0.0874	0.0895
Difference in %	-21.66%	-18.79%	-6.54%	-32.66%	-18.18%	-16.28%
Week	12					
Mean Return (USD)	2.42%					
Sharpe Ratio p.a.	0.1220					
Difference in %	14.17%					

Table 7: Trading Day Shift of 1Y Strategy

6 Conclusion

In this paper, we show the different characteristics of variance swaps. In the post crisis time, the implied volatility remains at a high level while the market is recovering and has a much smaller realized volatility. We also find that, by selling single variance swaps, one

[9] If a given date is not a business day, we shift them to the day after.

can improve the Sharpe ratio of the portfolio by achieving a much higher mean return. However, a pure selling strategy would also leverage the loss during the financial crisis and would not provide any diversification effect to the portfolio. We analyze a calendar spread strategy that consists of buying long term variance swaps and selling short term contracts at a higher frequency.

In a backtest we analyse a calendar spread strategy trading variance swaps to avoid the shortcomings encountered when trading single variance swap contracts. Ignoring bid-offer spreads, this calendar spread strategy gives us good protection from potential market crashes at almost zero or even negative costs. The combined strategy can maintain a correlation of almost -80% to the original portfolio, providing a noticeable diversification effect. We show that the volatility of a 70-30 equity-bond portfolio would decrease from 14.70% to 9.63% while the mean return would increase from 3.72% to 9.19%. Besides that, the calendar spread also greatly reduces the tail risk at a higher quantile. The loss at the 0.1% and 1% quantile would be less than 60% of the original portfolio. Finally, we link the notional of a variance swap to the portfolio value. We suggest that buying 1Y variance swaps at $3-4\%$ of the total value and selling 1M variance swaps at $15-20\%$ to the 1Y notional would provide the best performance of the calendar spread strategy.

Finally, we would like to point out that the results are based on a backtesting analysis and have no automatic predicitive power. For example, the strategy may perform badly in the future in case of a linear (zero variance) asset melt-down.

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