Foreign exchange derivatives

Commerzbank AG

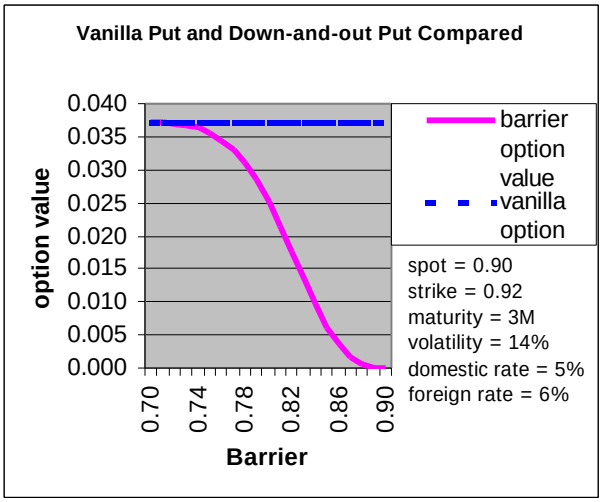
Isn't there anything cheaper than vanilla options? From an actuarial point of view a put or a call option is an insurance against falling or rising exchange rates, and surely a buyer would like to keep the premium at a minimal level. For this reason barrier options have been invented. They belong to the first generation exotics. The premium can be lowered by shifting risk to the option holder.
We give an overview of the issues related to barrier options

1. What is a barrier option

Varatio delectat - there are lots of different kinds of barrier options. A *standard barrier option* can be either a call or a put with the additional feature that the option becomes worthless if the spot hits a prespecified barrier. Such an option is called a *knock-out* call or knock-out put. Correspondingly there are options which are worthless unless and until the spot hits a barrier and at hitting time becomes a vanilla option (*knock-in* type). Clearly holding both a knock-in plus a knock-out and otherwise identical vanilla is the same as holding a mere vanilla option. Alternatively a long knock-out call can be replicated by a long call and a short knock-in call.
More generally, any option other than vanillas can have knock-out barriers. A knock-in type is often referred to as *kick-in* in the exotic case.

2. The popularity of barrier options

a) They are less expensive than vanilla contracts: in fact, the closer the spot is to the barrier, the cheaper the knock-out option. Any price between zero and the vanilla premium can be obtained by taking an appropriate barrier level. One must be aware however, that too cheap barrier options are very likely to knock out.



b) They allow to design foreign exchange risk exposure to special needs of customers. Instead of lowering the premium one can increase the nominal coverage of the vanilla contract by admitting a barrier. Several customers feel sure about exchange rate levels not being hit during the next month which could be exploited to lower the premium. Others really only want to cover their exchange rate exposure if the market moves drastically which would require a knock-in option.

c) The savings can be used for another hedge of foreign exchange

risk exposure if the first barrier option happened to knock out.

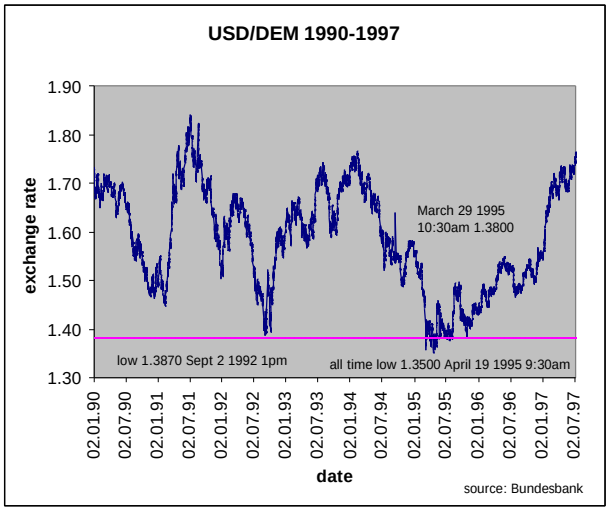
d) The contract is easy to understand if one knows about vanillas.

e) Many pricing and trading systems include barrier option calculations in their standard.

f) Pricing and hedging barriers in the Black-Scholes model is well-understood and most premium calculations use closed-form solutions which allow fast and stable implementation.

3. Barrier option crisis in 1994-96, questions about exotics in general

In the currency market barrier options became popular in 1994. The exchange rate between USD and DEM was then between 1.50 and 1.70. Since the all time low before 1995 was 1.3870 at September 2 1992 there were a lot of down and out barrier contracts written with a lower knock-out barrier of 1.3800. The sudden fall of the US Dollar in the beginning of 1995 came unexpected and the 1.3800 barrier was hit at 10:30 am on March 29 1995 and fell even more to its all time low of 1.3500 at 9:30 am on April 19 1995. Numerous barrier option holders were shocked that loosing the entire option is something that can really happen. The shock lasted for more than a year and barrier options had been unpopular for a while until many market participants had forgotten the event. Events like this often let the question about using exotics arise. Complicated products can in fact lead to unpleasant surprises. However, to cover foreign exchange risk in an individual design at minimal cost requires exotic options. Often they appear as an integral part of an investment portfolio. The number of market participants understanding the advantages and pitfalls is growing steadily.



4. Types of barriers

a) **American vs. European** - Traditionally barrier options are of American style, which means that the barrier level is active during the entire duration of the option: any time between today and maturity the spot hits the barrier, the option becomes worthless. If the barrier level is only active at maturity the barrier option is of European style and can in fact be replicated by *a vertical spread* and a *digital option*.

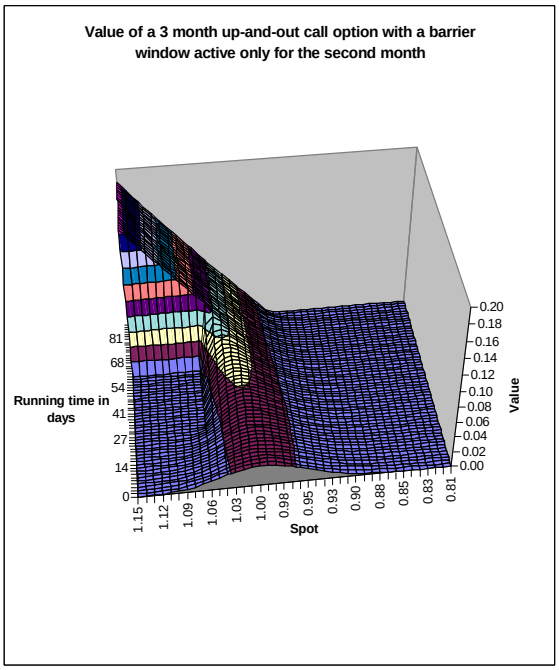
b) **Single, double and outside barriers** - Instead of taking just a lower or an upper barrier one could have both if one feels sure about the spot to remain in a range for a while. In this case besides vanillas, constant payoffs at maturity are popular, they are called *range binaries*. If the barrier and strike are in different exchange rates, the contract is called an *outside barrier option* or *double asset barrier option*. Such options traded a few years ago with the strike in USD/DEM and the barrier in USD/FRF taking advantage of the misbalance between implied and historic

correlation between the two currency pairs.

c) **Regular and reverse barriers** - Regular barrier options are *out-of-the-money* at hitting time, whereas reverse barrier options are *in-the-money* at that time. Loosing a reverse barrier option due to the spot hitting the barrier is more painful since the owner already has accumulated a positive intrinsic value. Hedging a reverse barrier option also causes difficulties due to large delta and gamma values.

d) **Rebates** - For knock-in options an amount R is paid at expiration by the seller of the option to the holder of the option if the option failed to kick in during its lifetime. For knock-out options an amount R is paid by the seller of the option to the holder of the option, if the option knocks out. The payment of the rebate is either at maturity or at the first time the barrier is hit. Including such rebate features makes hedging easier for reverse barrier options and serves as a consolation for the holder's disappointment. The rebate part of a barrier option can be completely separated from the barrier contract and can in fact be traded separately, in which case it is called a *one-touch (digital) option* or *hit* option (in the knock-out case) and *no-touch option* (in the knock-in case).

e) **Window or partial barriers** - Barriers need not be active for the entire lifetime of the option. One can specify arbitrary time ranges with piecewise constant barrier levels or even nonconstant barriers. *Linear and exponential barriers* are useful if there is a certain drift in the exchange rate caused, e.g., by a high interest rate differential (high swap rates).

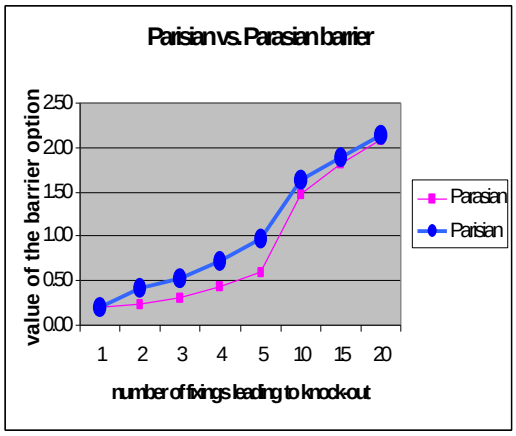


f) **Step and soft barriers** - Come on, the spot only crossed the barrier for a very short moment, can't you make an exception and not let my option knock out? This is a very common concern: how to get protection against price spikes. Such a protection is certainly possible, but surely has its price. One way is to measure the time the spot spends opposite the knock-out barrier and let the option knock out gradually. For instance one could agree that the options nominal is decreased by 10 % for each day the exchange rate fixing is opposite the barrier. This can be done linearly or exponentially. Such contracts are also referred to *as occupation time derivatives.*

g) **Fluffy barriers** - This is a way to let a barrier option knock out gradually not depending on the time spent beyond the barrier but the depth: For instance one can specify a barrier range of 2.20 to 2.30 where the option looses 25% of its nominal when 2.20 is breached, 50% when 2.25 is breached, 75% when 2.275 is

breached and 100% when 2.30 is breached.

h) **Parisian and Parasian barriers** - Another way to get price spike protection is to let the option knock out only if the spot spends a certain prespecified length of time opposite the barrier - either in total (Parasian) or in a row (Parisian). Clearly the plain barrier option is the least expensive, followed by the Parasian then the Parisian barrier option and finally the corresponding vanilla contract.

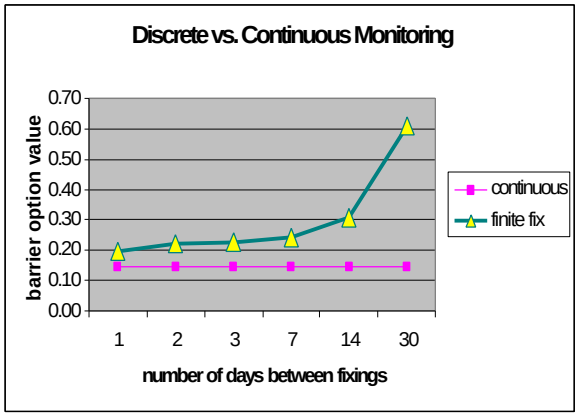


i) **Resetable barriers** - This is a way to give the holder of a barrier option a chance to reset the barrier during the life of the option n times at a priori determined N times in the future (N≥n). This kind of extra protection also makes the barrier option more expensive.

j) **Quanto barriers** - In foreign exchange options markets option payoffs are often paid in a currency different from the underlying currency pair. For instance a USD/JPY call is designed to be paid in EUR, where the exchange rate for EUR/JPY is determined a priori. Surely such features can be applied to barrier options as well.

5. How the barrier is monitored (continuous vs. discrete) and how this influences the price.

How often and when exactly do you check, whether an option has knocked out or kicked in? This question is not trivial and should be clearly stated in the deal. The intensity of monitoring can create any price between a standard barrier and a vanilla contract. The standard for barrier options is continuous monitoring. Any time the exchange rate hits the barrier the option is knocked out. An alternative is to consider just daily/weekly/monthly fixings which makes the knock-out option more expensive because chances of knocking out are smaller.



6. How breaching the barrier is determined

The *Foreign Exchange Committee* wishes to recommend to the foreign exchange community a *new set of best practices for the barrier options market*. In the next stage of this project, the Committee is planning to publish a revision to *the International Currency Options Master Agreement (ICOM) User Guide* to reflect the new recommendations.[1] Some key features are

- Name a *barrier determination agent*.

[1] For details see http://www.ny.frb.org/fxc/fxann000217.html

- Determination whether the spot has breached the barrier must be due to actual transactions in the foreign exchange markets.

- Transactions must occur between 5:00 A.M. Sydney time on Monday and 5:00 P.M. New York time on Friday.

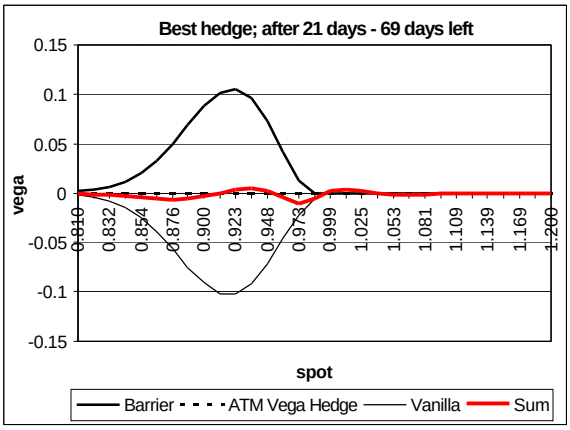
- Transactions must be of commercial size. In liquid markets, dealers generally accept that commercial size transactions are a minimum of $3 million.

- The barrier options determination agent may use cross-currency rates to determine whether a barrier has been breached in respect of a currency pair that is not commonly quoted.

7. Hedging methods, coping with high delta and gamma

Barrier options can be hedged statically with a portfolio of vanilla options. These approaches are problematic if the hedging portfolio has to be liquidated at hitting time, since volatilities for the vanillas may have changed between the time the hedge is composed and the time the barrier is hit. Moreover, the occasionally high nominals and low deltas can cause a high price for the hedge. For regular barriers a delta and vega hedge is more advisable. A vega hedge can be done almost statically using two vanilla options. In the example we consider a 3-month up-and-out put with strike 1.0100 and barrier 0.9800. The vega minimising hedge consists of 0.9 short 3-month 50 delta calls and 0.8 long 2-month 25 delta calls. Spot reference for EUR/USD is 0.9400 with rates 3.05% and 6.50% and volatility 11.9%.

Reverse barrier options have extremely high values for delta, gamma and theta when the spot is near the barrier and the time is close to expiry. This is because the intrinsic value of the option jumps from a positive value to zero when the barrier is hit. In such a situation a simple delta hedge is impractical. However, there are ways to tackle this undesirable state of affairs by moving the barrier or more systematically apply valuation subject to portfolio constraints such as limited leverage.



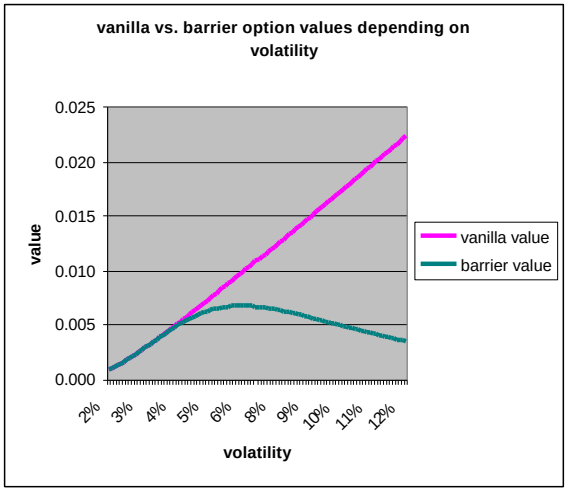
8. How large barrier contracts affect the market

Let's take the example of a reverse down-and-out put in EUR/USD with strike 0.90 and barrier 0.85. An investment bank delta-hedging a short position with nominal 10 Million has to buy 10 Million times delta EUR. As the spot goes down to the barrier, delta becomes larger and larger requiring the hedging institution to buy more and more EUR. This can influence the market since steadily asking for EUR slows down the spot movement towards the barrier and can in extreme cases prevent the spot from crossing the barrier. On the other hand, if the hedger runs out of breath or the downward market movement can't be stopped by the delta-hedging institution, then the options knocks out and the hedge is abandoned. Then suddenly fewer EUR are asked whence the downward movement of an

exchange rate can be accelerated once a large barrier contract in the market has knocked out.
The reverse situation occurs when the bank hedges a long position, in which case EUR has to be sold when the spot approaches the barrier. This can cause an accelerated movement of the exchange rate towards the barrier and a sudden halt once the barrier is breached.

9. Difference between market prices and theoretical Black-Scholes values - explained

Barrier options often do not trade at their theoretical Black-Scholes values. Vanilla options face the same problem, but since their price grows monotone in the volatility, one can change the volatility in the Black-Scholes formula and adjust the theoretical value to match the market price. It turns out that in foreign exchange option markets this volatility adjustment is typically positive for low delta puts and calls. This empirical phenomenon is called the volatility *smile*. It still allows to use the Black-Scholes formula for valuation, but one should be aware that the formula serves as a mean of communication, and the actually traded underlying is the volatility. However, the monotone volatility-price relationship does not hold for barrier options and thus can not be applied.



The price of barrier options is influenced by a variety of factors:

a) Smile of vanilla options - take the example of an upper knock-out barrier: if low-delta calls are more expensive than at-the-money calls one supposes that volatilities will grow with rising spot. This in turn increases the probability of knocking out and hence makes an up-and-out barrier option less expensive than the theoretical value. A systematic approach to take the vanilla smile into account is building a *Dupire-tree* or equivalently solving a Black-Scholes partial differential equation with a state- and time- dependent volatility coefficient, which ensures to match the observed vanilla option volatilities. However, this approach has not explained the way the market prices barrier options.
b) Price adjustments take the vega position of the barrier option into account.
c) The extreme values for delta and gamma in a dynamic hedging strategy described above can cause a further supplement to the price, particularly on the ask side.

10. Summary

We have seen that barrier options are an extremely versatile tool to manage foreign exchange exposure. Over the years lots of refinements to simple barrier options have been invented, such that market participants now use various ways to comply with customers' needs, expectations and risk profile.

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