1 Return distributions of equity-linked retirement plans with different capital guarantee mechanisms and fee structures

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1.1 Introduction

In the recent years an increasing demand for capital guaranteed equity-linked life insurance products and retirement plans has emerged. In Germany, a retirement plan, called Riester-Rente, is supported by the state with cash payments and tax benefits. Those retirement plans have to preserve the invested capital. The company offering a Riester-Rente has to ensure that at the end of the saving period at least all cash inflows are available. Due to the investors demand for high returns, banks and insurance companies are not only offering saving plans investing in riskless bonds but also in products with a high equity proportion. For companies offering an equity-linked Riester-Rente the guarantee to pay out at least the invested capital is a big challenge. Due to the long maturities of the contracts of more than 30 years it is not possible to just buy a protective put. Many different concepts are used by banks and insurance companies to generate this guarantee or to reduce the remaining risk for the company. They vary from simple Stop Loss strategies to complex dynamic hedging strategies. In our work we analyze the return distribution generated by some of these strategies. We consider several examples:

- A classical insurance strategy with investments in the actuarial reserve fund. In this strategy a large proportion of the invested capital is held in the actuarial reserve fund to fully generate the guarantee. Only the remaining capital is invested in products with a higher equity proportion.

The actuarial reserve fund is considered riskless. It usually guarantees a minimum yearly interest rate.

- A Constant proportion portfolio insurance strategy (CPPI) which is similar to the traditional reserve fund in that it ensures not to fall below a certain floor in order to generate the guarantee. In contrast to the traditional strategy the amount necessary to generate the guarantee is not fully invested in the riskless products. The amount invested in the more risky equity products is leveraged for a higher equity exposure. Continuous monitoring ensures that the guarantee is not at risk, since the equity proportion is reduced with the portfolio value becoming closer to the floor.
- A stop loss strategy where all the money is invested into pure equity until the floor is reached. If this happens all the invested capital is shifted into the riskless products in order to provide the guarantee at the end.

There are also equity-linked life insurance guarantees sold in Germany. In these products the insurance company promises to pay out the maximum of the invested amount and an investment in an equity fund reduced by a guarantee cost (usually yearly as a percentage of the fund value). The return distribution of these products highly depends on the guarantee cost. Due to the long maturities of the contracts, the pricing of this guarantee cost is not straightforward and the price is extremely dependent on the model which is chosen. For this reason they are not included in this comparative study. An introduction to equity-linked guarantees and their pricing can be found in (Hardy, 2003).

We simulate the return distribution for the different strategies and for different investment horizons. We analyze how fee structures, often used by insurance companies, affect the return distribution. We also study the impact of the cash payments of the state. Therefore we analyze an investment plan which maximizes the federal support.

To model the distribution we extend the jump diffusion model by Kou (Kou, 2002) by allowing the jumps to be displaced. Therefore, we go beyond the classical Black-Scholes model (Black and Scholes, 1973) and explicitly allow for jumps in the market as we could observe them within the last two years. One reason for using a jump diffusion model is that it better represents reality and therefore the rebalancing between equity and fixed income funds in the strategies under consideration is more realistic. The second reason is that Stop Loss and CPPI both only generate a sure guarantee in a market without jumps. Therefore, we analyze how often a CPPI and a Stop Loss strategy fail if we allow for jumps.

We find out that one of the driving factors of the return distribution is the fee structure of the contract. For some contracts only 85% of the invested capital is actually invested in the strategy. The remaining 15% is taken by the insurance or bank as sales and maintenance fees. With the yearly management fees of the underlying funds the return distribution is additionally weakened. Another very important factor of the return distribution is the guarantee concepts. Due to an equity exposure ranging from 36% to 100% percent the return distributions vary significantly.

1.2 The displaced double-exponential jump diffusion model

1.2.1 Model equation

As mentioned in Section 1.1, we would like to go beyond the classical Black-Scholes Model and allow for exponentially distributed jumps in the market. We extend the model by Kou (Kou, 2002) by allowing the jumps to be displaced.

The governing equation for the displaced jump diffusion model (DDE) is

$$\frac{\mathrm{d}S_t}{S_{t-}} = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}W_t + \mathrm{d}\left(\sum_{j=1}^{N_t}(V_j - 1)\right), \tag{1.1}$$

$$S_T = S_t \exp\left[\left(\mu - \frac{\sigma^2}{2} - \delta\right)\tau + \sigma W_{T-t}\right]\prod_{j=1}^{N_{T-t}} V_j, \tag{1.2}$$

where

(W_t) is a standard Brownian motion,

(N_t) a Poisson process with intensity $\lambda > 0$ and

V_j independent identically distributed random variables $V_j \sim e^Y$, where Y represents the relative jump size with a minimal jump of κ, therefore leading to jumps of Y in the range $(-\infty, -\kappa] \cup [\kappa, +\infty)$,

with parameters

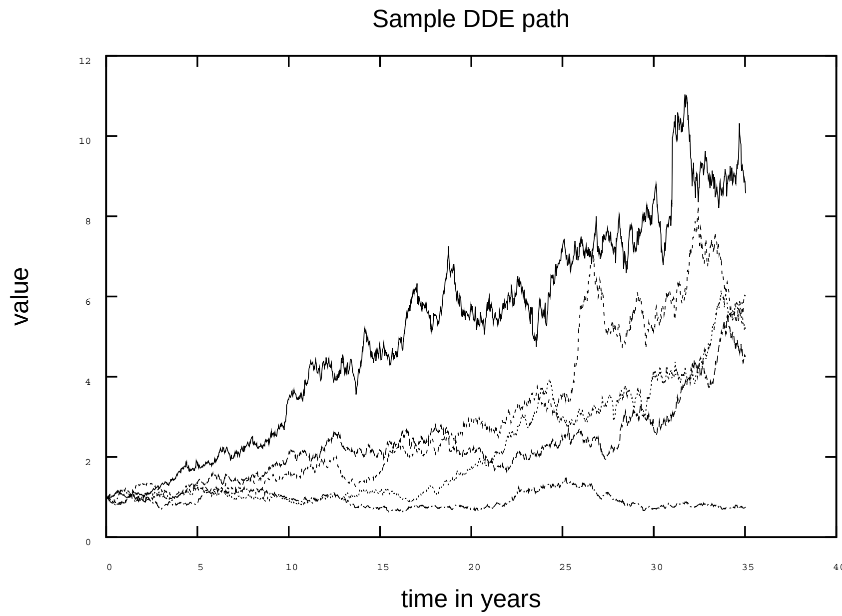

Figure 1.1: Displaced Double-Exponential jump process: simulated paths with parameters $T = 35$ years, $\mu = 6\%$, $\sigma = 14.3\%$, $\lambda = 5.209$, $\kappa = 2.31\%$, $\eta_1 = \eta_2 = \eta = 1/1.121\%$, $p = 0.5$, $S_0 = 1$.

- μ denoting the expected drift,
- σ denoting the volatility,
- λ denoting the expected number of jumps per year,
- δ the drift adjustment which is chosen such that the process S_t has the desired drift μ.

The processes (W_t), (N_t), and the random variables V_j are all independent.

We illustrate sample paths drawn from this distribution for a period of 35 years in Figure 1.1.

Except for the drift, the parameters are estimated to resemble the daily log returns of the MSCI World index for the last thirty years. For the drift we simulate different scenarios. We do not intend to simulate actively managed funds.

We assume a minimal jump size in order to distinguish between jumps arising from the Poisson process and the Brownian motion. The minimum jump size is chosen in order to qualify the 1% lowest and 1% highest daily log returns as jumps.

We chose the jumps Y to be exponentially distributed assuming only values outside the interval $(-\kappa, +\kappa)$.

Therefore, the jump part of the process has the density

$$f_Y(y) = \begin{cases} p\eta_1 e^{-(y-\kappa)\eta_1} & \text{if } y \geq \kappa, \\ 0 & \text{if } |y| < \kappa, \\ (1-p)\eta_2 e^{(y+\kappa)\eta_2} & \text{if } y \leq -\kappa, \end{cases} \tag{1.3}$$

with $\eta_1 > 1$, $\eta_2 > 0$ and $0 \leq p \leq 1$.

1.2.2 Drift adjustment

Similar to the work of Kou (Kou, 2002) we calculate the drift adjustment for the jump process by

$$\begin{aligned} \delta &= \mathrm{E}[e^Y - 1] \\ &= \lambda \left(p\eta_1 \frac{e^{+\kappa}}{\eta_1 - 1} + (1-p)\eta_2 \frac{e^{-\kappa}}{\eta_2 + 1} - 1 \right). \end{aligned} \tag{1.4}$$

In this paper we use $\eta = \eta_1 = \eta_2$ and $p = 0.5$.

1.2.3 Moments, variance and volatility

The variance of the random number $\ln \frac{S_t}{S_0}$ of the process (1.2) can be written as

$$\begin{aligned} \mathrm{Var}\left[\ln \frac{S_t}{S_0}\right] &= \sigma^2 t + \mathrm{Var}\left[\sum_{k=1}^{N_t} U_k(\kappa + H_k)\right] \\ &= \sigma^2 t + \lambda t((\kappa + h)^2 + h^2), \end{aligned}$$

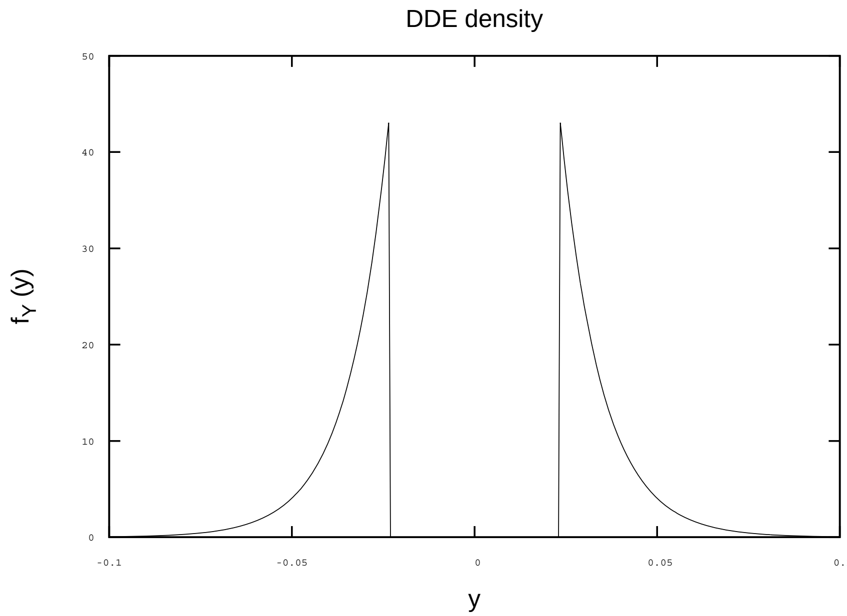

Figure 1.2: Displaced Double-Exponential density of Y with parameters $\kappa =$ 2.31%, $\eta_1 = \eta_2 = \eta$ =1/1.121%, $p = 0.5$

where the H_k are independent exponentially distributed random numbers with expectation $h = \frac{1}{\eta}$. U_k is a random variable which takes the value of +1 and −1 with probability $\frac{1}{2}$. We calculate the first two moments

$$\begin{aligned}
& \mathrm{E}\left[\sum_{k=1}^{N_t} U_k(\kappa + H_k)\right] \\
= & \sum_{n=0}^{\infty} \mathrm{E}\left[\sum_{k=1}^{n} U_k(\kappa + H_k)\right] \cdot \mathrm{P}[N_t = n] \\
= & \sum_{n=0}^{\infty} n \cdot 0 \cdot \mathrm{P}[N_t = n] \\
= & \ 0,
\end{aligned}$$

$$\begin{aligned}
& \mathrm{E}\left[\sum_{k=1}^{N_t} U_k(\kappa + H_k)\right]^2 \\
= & \sum_{n=0}^{\infty} \mathrm{E}\left[\sum_{k=1}^{n} U_k(\kappa + H_k)\right]^2 \cdot \mathrm{P}[N_t = n] \\
= & \lambda t((\kappa + h)^2 + h^2).
\end{aligned}$$

For the variance we obtain

$$\begin{aligned}
& \mathrm{Var}\left[\sum_{k=1}^{N_t} U_k(\kappa + H_k)\right] \\
= & \mathrm{E}\left[\sum_{k=1}^{N_t} U_k(\kappa + H_k)\right]^2 - \left(\mathrm{E}\left[\sum_{k=1}^{N_t} U_k(\kappa + H_k)\right]\right)^2 \\
= & \lambda t((\kappa + h)^2 + h^2).
\end{aligned}$$

Finally, the volatility of the DDE-process is

$$\sqrt{\frac{1}{t}\mathrm{Var}\left[\ln \frac{S_t}{S_0}\right]} = \sqrt{\sigma^2 + \lambda((\kappa + h)^2 + h^2)}. \tag{1.5}$$

1.3 Parameter estimation

1.3.1 Estimating parameters from financial data

We estimate the parameters based on the historical data of the **MSCI Daily TR (Total Return) Gross (gross dividends reinvested) in USD** for the period between January 1 1980 and October 2 2009. We denote these prices with $x_0, x_1, \ldots, x_N$ and the log-returns by

$$r_i \triangleq \ln \frac{x_i}{x_{i-1}}, \quad i = 0, 1, \ldots, N. \tag{1.6}$$

Parameter	Value
Total volatility $\hat{\sigma}_{tot}$	14.3%
Volatility of the diffusion part $\hat{\sigma}$	11.69%
Jump intensity λ	5.209
Minimum jump size κ	2.31%
Expected jump size above minimum jump size h	1.121%
Drift adjustment δ	0.339%

Table 1.1: Estimated parameters for the DDE-process.

The estimate for the daily log return is

$$\bar{r} \quad = \quad \frac{1}{N}\sum_{i=1}^{N} r_i. \tag{1.7}$$

The estimate for the total volatility $\hat{\sigma}_{tot}$ is

$$\hat{\sigma}_{tot}^2 \quad = \quad \frac{\#\text{Prices per year}}{N-1}\left(\sum_{i=1}^{N} r_i^2 - N\bar{r}^2\right). \tag{1.8}$$

To determine the parameters for the jump process we have to define a level κ such that r_i with $\|r_i\| \geq \kappa$ is considered to be a jump. To determine this κ we define for a given level $u \in [0,1]$ the u% smallest and u% biggest daily log returns The level u should be chosen such that the resulting returns are intuitively considered as jumps. If u is chosen too high, even small log-returns are considered jumps, and if u is too low, almost no jumps occur. Of course, this level is subjective. We have chosen $u = 1\%$ because in this case only daily changes of more than 2% are considered to be a jump. Changes of less than 2% can be explained with the diffusion part with sufficiently high probability. It turns out that for the analyzed MSCI World Index, the smallest up-jump and the smallest down-jump is almost equal. The absolute values of the relative jump size is on average 2.31%. We use this minimal jump size as an estimator for κ The value η of the single parameter exponential distribution is chosen such that η fits the mean of the observed jumps. From the financial data and the already fixed parameters we obtain $h = 1/\eta$ with $\eta = 1.21\%$. The number of jumps divided by the total number of observations yields an estimate for the jump frequency. Annualizing this frequency we can estimate λ to be 5.21.

Finally we have to correct the estimator for the volatility according to Equation (1.5) since the volatility consists of the jump part and the diffusion part.

We summarize the estimated parameters in Table 1.1.

1.4 Interest rate curve

To calculate the current value of the future liability (floor) and the performance of the riskless investments, we use the zero bond curve as of October 1 2009. The curve is extracted from the money market and swap rate quotes on Reuters. This curve is static and not simulated. Interpolation is done linear in the rates.

See Table 1.2 for the calculated discount factors and the market quotes as seen on Reuters.

1.5 Products

In this section we describe the analyzed products and the assumptions we made.

1.5.1 Classical insurance strategy with investment in the actuarial reserve fund

The current value of the future liability is calculated and a sufficient amount to meet this liability in the future is invested in the actuarial reserve fund. The actuarial reserve fund is assumed to be riskless and accrues the interest implied by the current zero bond curve but at least the currently guaranteed interest of 2.25%. The return participation is added to the contract once a year. Only the excess amount which is not needed for the guarantee is invested in the risky asset. We assume that the calculation of the amount needed to meet the future liability is based on the guaranteed interest rate of 2.25%. See Figure 1.3 for a picture of the exposure distribution in the classical insurance case.

Date	Instrument	Rate	Discount factor
11/3/2009	Money market	0.43%	99.96%
12/3/2009	Money market	0.59%	99.89%
1/4/2010	Money market	0.75%	99.80%
2/3/2010	Money market	0.84%	99.71%
3/3/2010	Money market	0.92%	99.61%
4/5/2010	Money market	1.01%	99.48%
5/3/2010	Money market	1.06%	99.37%
6/3/2010	Money market	1.10%	99.26%
7/5/2010	Money market	1.14%	99.13%
8/3/2010	Money market	1.18%	99.01%
9/3/2010	Money market	1.20%	98.88%
10/4/2010	swap	1.20%	98.80%
10/3/2011	swap	1.71%	96.64%
10/3/2012	swap	2.15%	93.76%
10/3/2013	swap	2.46%	90.64%
10/3/2014	swap	2.71%	87.30%
10/5/2015	swap	2.92%	83.85%
10/3/2016	swap	3.15%	80.11%
10/3/2017	swap	3.24%	77.01%
10/3/2018	swap	3.36%	73.71%
10/3/2019	swap	3.46%	70.47%
10/5/2020	swap	3.55%	67.25%
10/4/2021	swap	3.64%	64.09%
10/3/2022	swap	3.72%	61.07%
10/3/2023	swap	3.79%	58.18%
10/3/2024	swap	3.84%	55.43%
10/3/2029	swap	3.99%	44.76%

Table 1.2: Extracted discount factors from money market quotes and swap rates.

1.5.2 Constant proportion portfolio insurance (CPPI)

The Constant proportion portfolio insurance structure (CPPI) works similar to the classical strategy of investments in the actuarial reserve fund. The difference is that instead of investing only the excess amount into the risky asset the excess amount is leveraged in order to allow for a higher equity participa-

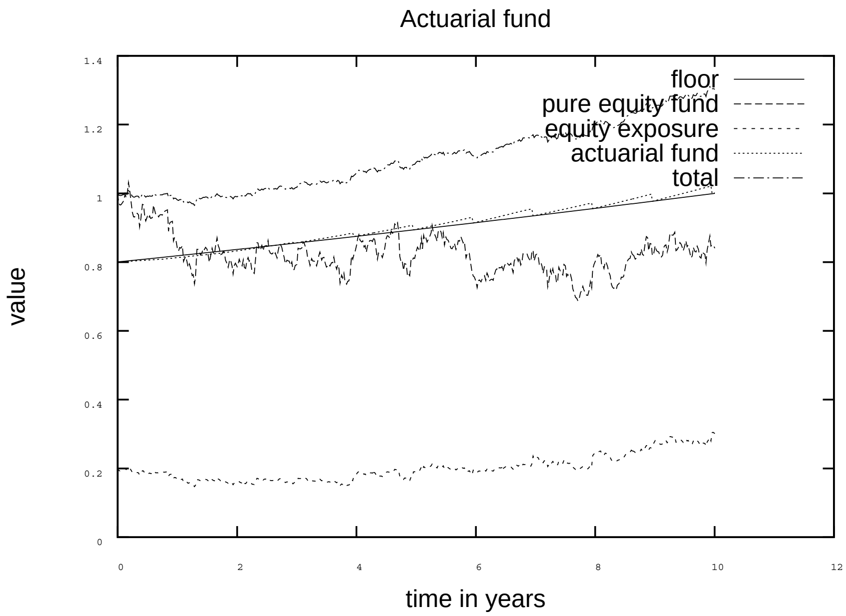

Figure 1.3: Simulated path for a classical insurance strategy and 10 year investment horizon

tion. The investment is monitored on a continuous basis to guarantee that the investment doesn't fall below the floor. With F being the floor of the future obligations, NAV the net asset value of the fund and a the leverage factor, the rebalancing equation for the risky asset R is

$$R = \max\left(a\left(NAV - F\right), NAV\right). \tag{1.9}$$

The leverage factor determines the participation on the equity returns. The higher the leverage factor a, the higher the participation on positive return but also the risk to reach the floor. For a leverage factor of $a = 1$ the structure becomes static when assuming constant interest rates. A commonly used value for a in the industry is 3. For a sample path of a CPPI and the equity and fixed income distribution structure with leverage factor 3 see Figure 1.4. This strategy guarantees 100% capital protection in a continuous model. In a model with jumps this is not the case anymore. The strategy is subject to gap risk,

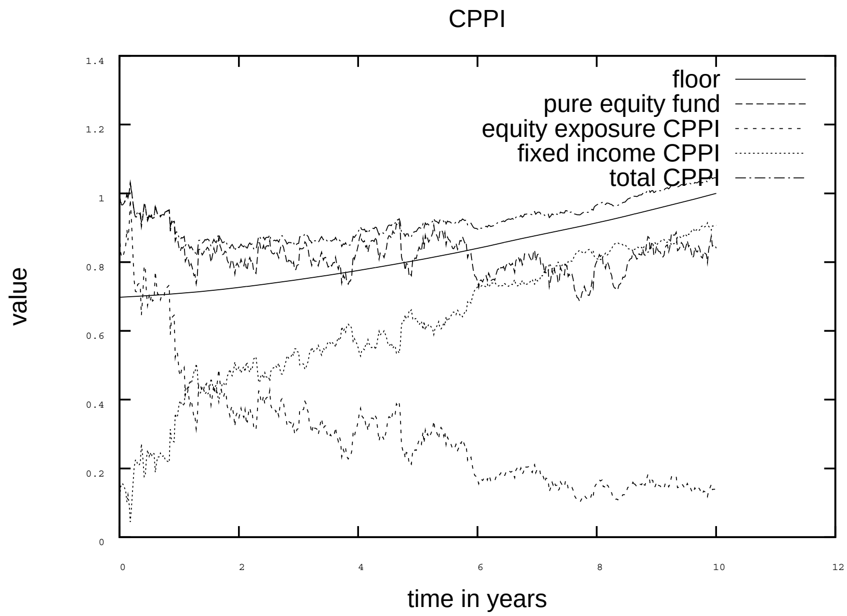

Figure 1.4: Simulated CPPI path with leverage factor 3 and 10 years investment horizon

the risk that due to a jump in the market, rebalancing is not possible and the fund value drops below the floor. We neglect liquidity issues here which would cause an additional risk. However due to the continuous reduction of the equity exposure this risk is rather small. See for example (Black and Perold, 1992) for the general theory of constant proportion portfolio insurance.

1.5.3 Stop loss strategy

In the stop loss strategy 100% of the equity amount is held in the risky fund until the floor is reached. In this case all the investment is moved to the fixed income fund to generate the guarantee at maturity. See Figure 1.5 for a path where the stop loss barrier is reached and all the investment is shifted into the fixed income fund. This strategy is riskless as the CPPI strategy in a continuous

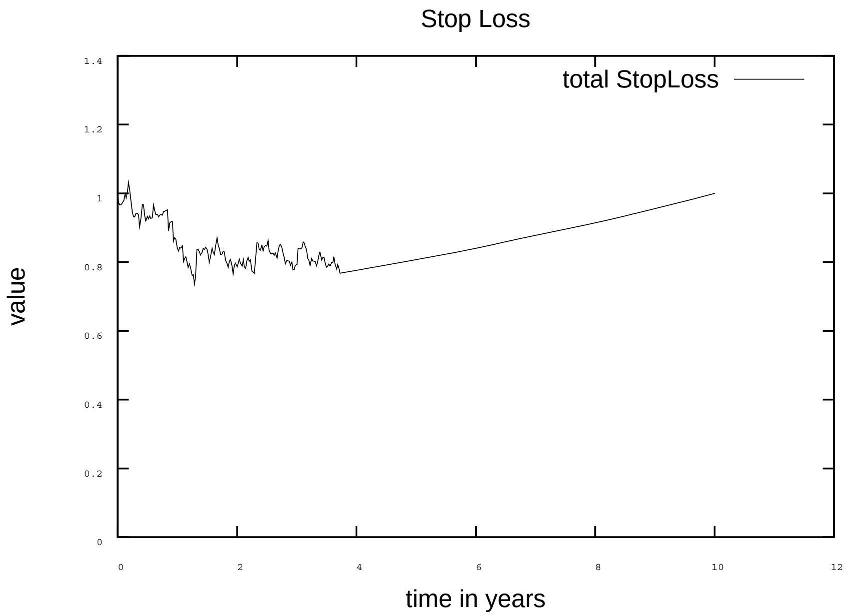

Figure 1.5: Simulated stop-loss path with 10 years investment horizon

model. In a model with jumps again we are imposed to gap risk. We neglect liquidity issues here which actually forces the insurer to liquidate the risky asset before it reaches the floor level. This issue becomes especially important for large funds, which is often the case for retirement plans or life insurance products since the amount to liquidate is so big that it actually influences the market.

1.6 Payments to the contract and simulation horizon

Since capital guaranteed life insurance products are especially popular in Germany under the master agreement of the Riester-Rente, we consider a typical payment plan with an horizon of 20 years. To be eligible for the maximal amount of cash payments and tax benefits the insured has to spend at least 4% of his yearly gross income for the insurance product, including the payments from the state but no more than 2,100 Euro. In this case he receives cash payments of 154 Euro per year, and additional 185 Euro for each child born before January 1 2008 and 300 Euro for each child born on or after this date. Even though this is not the focus of this paper, we assume a saving plan that allows for these benefits in order to compare typical cost structure against the state benefits. We consider the situation of a person being 45 years old when entering the contract and earning 30,000 Euro a year. We further assume that he has one child born after January 1 2008 but before entering the contract. In this case the insured receives 454 Euro from the state, so he actually only has to pay 746 Euro per year to reach 1,200 per year (4% of his income). This is a very high support rate of 37%. For comparison, if we take an investor without children, earning 52,500 Euro per year, the support rate would only be maximal 7.3%. We assume a monthly payment of 100 Euro and do not distinguish between payments made by the state and by the insured. The total nominal amount is $20 \times 1,200 = 24,000$ Euro. This is the amount the issuer of the plan needs to guarantee at retirement. There is no guarantee during the lifetime of the contract. Especially in the case that the insured dies before retirement, the payments to the contract are not guaranteed and only the current account value can be transfered to another contract or payed out. In case the contract is payed out, payments form the state will be claimed back.

1.7 Cost structures

We study the impact of different cost structures which are often seen in insurance products. Often the fee structure of these products is rather complex and consists of a combination of various fees.

- Sales and Distribution cost: These costs are usually charged to pay a sales fee for the agent who closed the deal with the insured. These fees are usually dependent on the total cash contracted to pay into the contract until maturity. However they are usually charged uniformly distributed over the first 5 years of the contract. In insurance business they are called α-cost.
- Administration cost: These costs are usually charged on the cash payments to the contract during the entire lifetime of the contract. They are usually charged to cover administrative costs of the contract. In insurance

Scenario	drift μ	volatility σ_{tot}
Standard	6%	14.3%
Optimistic	8%	14.3%
Pessimistic	4%	14.3%

Table 1.3: Model parameters for the different scenarios. The other model parameters are takes from the estimates in Table 1.1

business they are called β-cost.

- Capital management cost: These costs are charged based on the sum of the payments up to the effective date. They are usually charged for capital management.

To compare the impact of these different cost structures we analyze costs that are equivalent in terms of the current value. We assume a total fee of 4% of all payments to the contract, i.e. 960 Euro. The current value of these fees with the applied zero bond curve is 681 Euro. Since the typical costs in insurance products are usually a combination of all these fees we also simulate the impact of the sum of these single fees which is a commonly used cost charge.

1.8 Results without costs

We present the simulation results for different scenarios. Since the drift is hardly estimated from the past realization, we calculate the results for three different drift assumptions.

In Figure 1.6 it can be seen how the distribution varies between the different strategies.

The stop loss strategy has an expected distribution very close to the pure equity investment since it has the highest equity participation as can be seen in Table 1.4,Table 1.5 and Table 1.6. So, for the bullish investor this might be the optimal investment for his Riester-Rente. A similar return profile is provided by a CPPI structure with a high leverage factor. The advantage of the CPPI product in practice is that due to the continuous reduction of the exposure if the market is performing badly, the liquidity issue is smaller than for the stop loss strategy. However, as can be seen in Figure 1.6, the risk

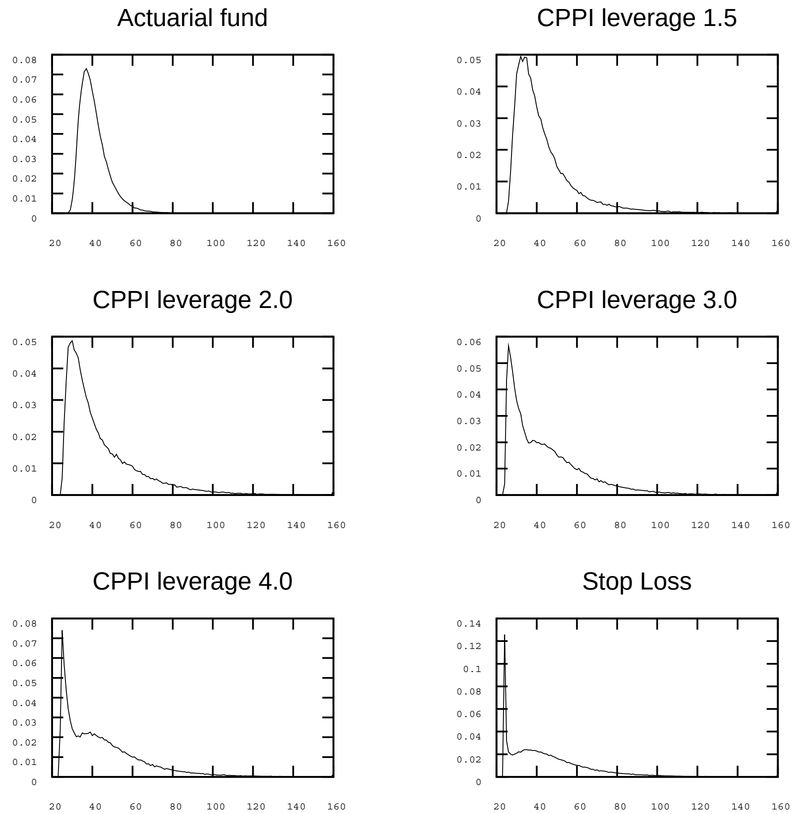

Figure 1.6: Return distribution of the different strategies. We list the capital available at retirement (in units of 1,000 EUR) on the x-axes.

of returns close to zero is rather high for both, the stop loss and the CPPI with a high leverage factor. For the bearish investor a classical product with an investment mainly in the actuarial fund or a CPPI product with a small leverage factor could be the better choice.

Strategy	mean	median	exposure
Actuarial	40945	39393	36.54%
CPPI Leverage 1.5	43636	38826	73.48%
CPPI Leverage 2	44731	38129	87.83%
CPPI Leverage 3	45211	39726	94.03%
CPPI Leverage 4	45326	40489	95.56%
Stop Loss	45443	40867	96.65%

Table 1.4: Results in the standard scenario.

Strategy	mean	median	exposure
Actuarial	45055	42991	38.91%
CPPI Leverage 1.5	53074	45937	79.01%
CPPI Leverage 2	55850	48409	92.66%
CPPI Leverage 3	56765	50825	97.14%
CPPI Leverage 4	56942	51126	98.06%
Stop Loss	57059	51252	98.69%

Table 1.5: Results in the optimistic scenario.

Strategy	mean	median	exposure
Actuarial	37706	36541	34.25%
CPPI Leverage 1.5	37160	34041	67.81%
CPPI Leverage 2	36892	32332	81.77%
CPPI Leverage 3	36698	30838	89.19%
CPPI Leverage 4	36640	30751	91.32%
Stop Loss	36693	32118	92.96%

Table 1.6: Results in the pessimistic scenario.

1.9 Impact of costs

It can be seen that the fees have a high impact on the return distribution. Even if the fees have actually the same current value, the impact on the return distribution is different. The alpha cost weakens the expected return most since

Cost	mean	median	exposure
No cost	40945	39393	36.54%
Alpha cost	38923	37678	30.89%
Beta cost	39120	37747	33.83%
Cost on accumulated payments	39231	37782	35.17%
Sum of all fees	35387	34422	25.93%

Table 1.7: Results for the actuarial reserve product in the standard scenario with costs.

Cost	mean	median	exposure
No cost	45326	40486	95.56%
Alpha cost	43236	38423	94.36%
Beta cost	43433	38496	94.42%
Cost on accumulated payments	43546	38566	94.51%
Sum of all fees	39506	33966	91.13%

Table 1.8: Results in the CPPI strategy with leverage factor 3 with costs in the standard scenario.

it decreases the exposure at the beginning of the saving period. This impact is very high for the actuarial reserve product which even without fees only has an average equity participation of 36.54%. The alpha fee reduces this further to 30.9% as can be seen in Table 1.7. The fees on the accumulated payments has the least impact since these are mainly charged at the end of the saving period and therefore the impact on the equity exposure is smaller. The insured has to carefully study whether the negative impact of the fee structure is actually fully compensated by the federal cash payments. This highly depends on the cost structure which varies massively between the different products and on the income and family situation of the insured, which in turn determines the amount of cash benefits from the state. In many cases it may be advisable to choose a product outside the class of Riester-Rente that has a smaller cost ratio. In this case the investor can buy less costly products and can freely choose a product without capital protection, which has a higher expected return. A more detailed analysis of the different cost structures can be found in (Detering, Weber and Wystup, 2009).

Cost	mean	median	exposure
No cost	45443	40869	96.65%
Alpha cost	43376	38918	95.62%
Beta cost	43577	39044	95.67%
Cost on accumulated payments	43706	39139	95.75%
Sum of all fees	39747	35141	92.83%

Table 1.9: Results in the stop loss strategy with costs in the standard scenario.

1.10 Impact of jumps

The CPPI and the Stop Loss strategy are risk free in a continuous equity model. However, in a model with jumps, we are exposed to *gap risk*, which means that the value of portfolio of risky assets can fall below the floor. In this case the strategy fails to generate the guarantee. In practice the leverage factor is chosen such that even a very big jump in the market still maintains the guarantee. For example, a 20% jump in the market doesn't cause a loss for a rebalanced portfolio if the leverage factor is below 5. In this case all the equity exposure would be lost, but the guarantee could exactly be generated. So, with a moderate leverage factor the remaining gap risk is negligible. This is also reflected in our jump diffusion model. For the stop loss strategy the situation is different since the equity exposure does not decrease when approaching the floor level. We calculate how often the strategy fails in case of a jump on average. The expected number of guarantee shortfalls is shown in Table 1.10. We assume that after the stop loss level is reached and all money is invested in the risk free fund it stays there until maturity. Only new cash flows are again invested in the risky fund. Therefore, it might happen that there are several guarantee shortfalls in one path. The shortfall number is the accumulated sum of shortfalls over one path. We show the number of shortfalls for a contract without fees and for a contract with fees. The fees increase the risk of a shortfall since the insurance company has to guarantee the cash payments made by the insured.

Table 1.10 shows that in the CPPI strategy the guarantee is never at risk. The shortfalls in the situation with fees are actually not caused by large jumps, but are caused by high fees close to maturity. At this time the fee charge on the incoming cash flow is so high compared to the difference between invested

Product	number of paths with gap	average realized gap
CPPI, factor 3, no fees	0	0
CPPI, factor 3, with fees	265	50
Stop loss, no fees	18804	219
Stop loss, with fees	31317	354

Table 1.10: Shortfalls for stop loss and CPPI for 100,000 simulations.

amount and its present value that the cash contribution does not suffice to ensure the guarantee. In case of a low performing path history the investments done earlier are not able to compensate for this.

1.11 Summary

We have compared the performance of savings plans within the class of difference capital guarantee mechanisms: from the stop loss to classic investments in actuarial reserve funds. CPPI strategies with different leverage factors can be viewed as a compromises between these two extremes. In bullish markets savings plans with a high equity ratio perform the best, in bearish markets the classic insurance concept shows better returns. A stop loss strategy suffers from gap risk, whence a CPPI strategy combines the strength of both gap risk minimization and equity ratio maximization. The effect of fees on the savings plans dominates the performance, especially in typical fee structures found in the German Riester-Rente. The private investor is advised to check carefully if the federal cash payments can compensate the fees taking into account his own salary and tax situation.

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